112-2 Statistics (II)

Quiz 1 Solution

Spring 2024

1. (b)
2. (d)
3. The level of significance is the probability of making a **Type I error** when the null hypothesis is **true as an equality**.
4. The **critical value** is the value of the test statistic that corresponds to an area of α in the lower tail of the sampling distribution of the test statistic.
5. In hypothesis testing, the p-value is the probability of obtaining test results at least as extreme as the result actually observed, under the assumption that the null hypothesis is correct.
6. A small p-value means that such an extreme observed outcome would be very unlikely under the null hypothesis.
7. (a)

$$H_0 : \mu \geq 9$$
$$H_1 : \mu < 9$$

(b)
$t_{0.01}(84) = 2.3715, \quad t_{0.005}(84) = 2.6356$

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{7.27 - 9}{6.38/\sqrt{85}} = -2.5$$

p-value is between 0.005 and 0.01 by using t table.

Exact p-value is

$$P(T < -2.5) = a$$

$$\frac{a - 0.01}{0.005 - 0.01} = \frac{2.5 - 2.3715}{2.6356 - 2.3715} \quad \Rightarrow a \approx 0.0075$$

(c)
Because p-value is less than $\alpha = 0.01$, we reject null hypothesis. The mean tenure of a CEO is significantly lower than nine years. The claim of the shareholders group is not valid.

8. (a)
Accepting null hypothesis and letting the process continue to run when actually overfilling or underfilling exists.
(b)
Rejection Region:

$$\{\bar{X} < k_1,\ \bar{X} > k_2\}$$

where

$$k_1 = 16 - z_{0.025}\frac{0.8}{\sqrt{30}} = 15.7137$$
$$k_2 = 16 + z_{0.025}\frac{0.8}{\sqrt{30}} = 16.2862$$

For $\mu = 16.5$,

$$\beta = P\left(\frac{15.7137 - 16.5}{0.8/\sqrt{30}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{16.2862 - 16.5}{0.8/\sqrt{30}}\right) = 0.0749$$

so power of test when the machine is overfilling by 0.5 ounces equals
$1 - \beta = 1 - 0.0749 = 0.9251$

9. (i)
There is a duality between confidence intervals and hypothesis tests.
Let $X_1, \cdots, X_n$ be a random sample from a normal distribution having unknown mean μ and known variance σ^2. We consider testing the hypotheses

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu \neq \mu_0$$

Consider a test at a specific level α that rejects for $\{|\bar{X}| > k\}$, where k is determined so that $P(|\bar{X}| > k) = \alpha$ if H_0 is true.

The test thus accepts when

$$\mu_0 - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu_0 + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

A $100(1-\alpha)\%$ confidence interval for μ_0 is

$$\left[\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}},\ \ \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right]$$

Comparing the acceptance region of the test to the confidence interval, we see that μ_0 lies in the confidence interval for μ if and only if the hypothesis test accepts.

(ii)

Constructing a $100(1-\alpha)\%$ confidence interval and rejecting *H₀* whenever the interval does not contain μ_0 is equivalent to conducting a two-tailed hypothesis test with α as the level of significance.