Local Regression With Meaningful Parameters

Rafael A. Irizarry

Prepared by Je-Wei Lin

Department of Statistics, National Chengchi University

Outlines

- Introduction
- Local Regression
- Local Harmonic Regression
- Extensions
- Conclusion

Objectives

- Using local regression to explore sound signals.
- Using harmonic function to fit the smooth data.
- The fitted parameters frequency and amplitudes which contain meaningful interpretations.

Concept

- Assume a model without making parametric assumptions about the mean structure as

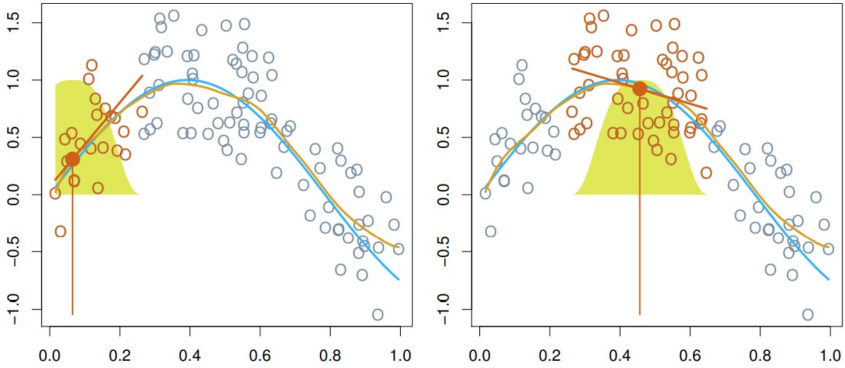
$$Y_i = f(\boldsymbol{x_i}) + \epsilon_i \tag{1}$$

- At each point x_0, use regression function to fit nearest neighbors of that point.

Procedure

- Select a window and define a weight function.
- Use the weighted average points to estimate fitted value.
- Alter the different x_0 and repeat the procedure.

Local regression illustrated on simulated data



Notations

- Using $x_i = t_i$ to denote the time associated with the ith measurement y_i.
- Let h denote the span or window size which controls the **smoothness**.
- A weight W decreases when the distance Δ increases.

Method

- Consider tri-cube weighting function

$$W\left(|t_i - t_0|, h\right) \tag{2}$$

$$W\left(\Delta, h\right) = \left\{1 - \left(\Delta/h\right)^3\right\}^3 \tag{3}$$

- Using WLS to estimate β

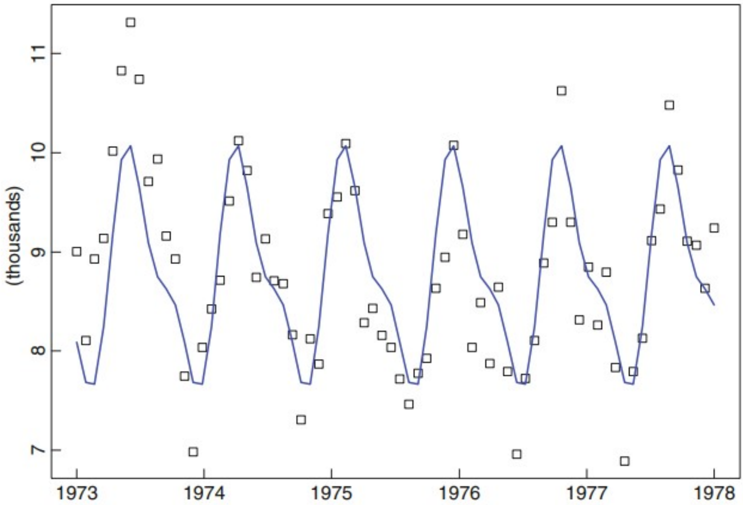
$$argmin \sum_{i=1}^{n} w_i\left(t_0\right)\left\{y_i - s\left(t_i; \beta\right)\right\}^2 \tag{4}$$

Harmonic Regression

- Many time series are influenced by seasonally varying factors, the effect of which can be modeled by a periodic component.

$$s_t = a_0 + \sum_{j=1}^{K} (a_j cos(\lambda_j t) + b_j sin(\lambda_j t)) \tag{5}$$

Example - Accidental Deaths



Lohess

- Using harmonic function to approximate local signal instead of polynomials for $s(t_i; \beta)$.
- Define the method of local harmonic regression as **lohess**.

Terminology

- **Fundamental Frequency** is defined as the lowest frequency of a periodic waveform.
- **Harmonics** is a sinusoidal wave with a frequency that is a positive integer multiple of the fundamental frequency of a periodic signal.

Terminology (Cont'd)

- **Partial** is any of the sine waves of which a complex tone is composed, not necessarily with an integer multiple of the lowest harmonic.

Music Sound Signals

- Musical instruments' sounds can be characterized by a periodic signal plus stochastic noise model.
- Examing spectrograms to argue signal produced, a C-note or D-note is periodic.
- We define $\lambda = n/p$ as frequency and its units are cycles per every n measurements.

Music Sound Signals (Cont'd)

$$s\left(t_i;\beta\right) = \mu + \sum_{k=1}^{K}\left\{a_k cos\left(\frac{2\pi k}{n}\lambda i\right) + b_k sin\left(\frac{2\pi k}{n}\lambda i\right)\right\} \quad (6)$$

$$= \mu + \sum_{k=1}^{K}\rho_k cos\left(\frac{2\pi k}{n}\lambda i + \phi_k\right) \quad (7)$$

- where $\rho_k = \sqrt{a_k^2 + b_k^2}$ and $\phi_k = \arctan(\frac{-b_k}{a_k})$

Meaningful Parameters

- With **lohess**, the fitted parameters can be interpreted as local mean level $\hat{\mu}$, frequency λ, amplitudes $\hat{\rho}_k$.
- The fitted parameters may be considered more informative than the smooth f.

Example - Timbre Morphing

- Timbre modification is the process of changing a sound by manipulation musically parametric representations.
- Creating the sound $s(t_i; m(\hat{\beta}))$ to modify the oboe sound through parametric representation and modulates the amplitudes of the even harmonics.

Asymptotic Properties

- Consider a locally periodic signal can be expressed as

$$Y_i = s\left(t_i; \beta\left(t_i\right)\right) + \epsilon_i \tag{8}$$

- We can find the estimates with lohess have asymptotic normality properties if
  - The sample rate n is high enough.
  - $\beta\left(t_i\right)$ is smooth enough.
  - Fundamental frequency is large enough.

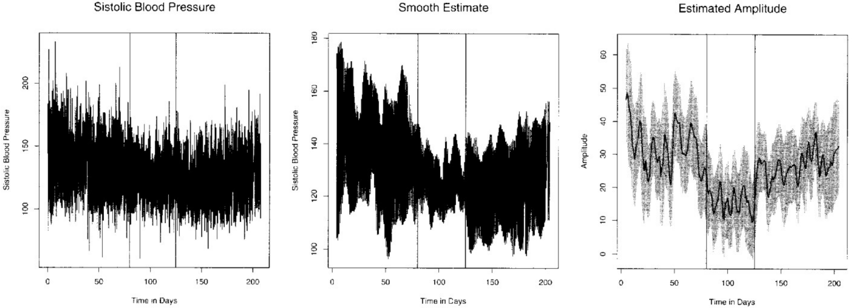
Asymptotic Properties (Cont'd)

- For the music signals, we have enough oscillation to have the harmonic model as good approximation and enough data points to use largr sample approximations for the sample rate and frequency.

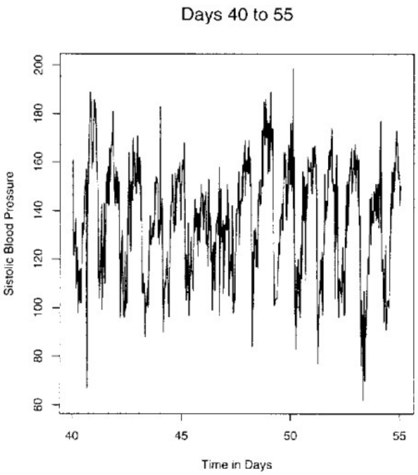
Application - Physiology

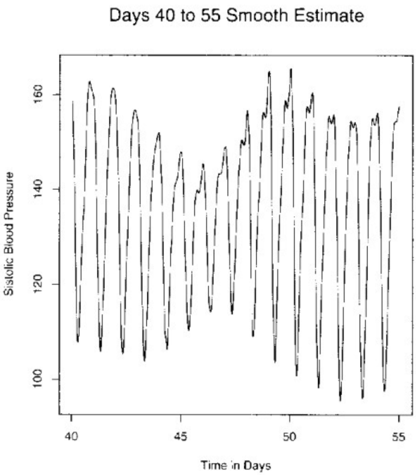
- Systolic blood pressure taken on a patient that suffers from a condition dubbed circadian hyper amplitude tension(CHAT).
- Oberserve the fluctuaion of pressure after taking nifedipine and blocalcin which follows circadian pattern.

Example - Systolic Blood Pressure



Example - Systolic Blood Pressure (Cont'd)





Coments

- If we believe the shape of the oscillations during days that are close together are more similar than for days that are far apart, then using smaller window sizes is an alternative that can improve the local approximation.

Extensions and Conclusion

- Nonpolynomial local models can be useful in other datasets where functions other than polynomial or harmonic may be used for $s\,(t_i;\beta)$.(e.g. DJIA)
- Lohess is useful for smoothing time series with periodic trend and serve as exploratory analysis tools.