2020/12/07 微積分小考(3) - §3.9~§4.5　滿分為 100 分

整體批改標準：符號標錯扣 2 分，說明不清楚、不完整扣 3 分，
過程沒寫或寫錯扣該題分數一半。

1. (10%)

$$f(x)=x^{\frac{1}{3}} \Rightarrow f'(x)=\frac{1}{3}x^{\frac{-2}{3}}=\frac{1}{3x^{2/3}}$$

$$\Rightarrow L(x)=f'(-8)(x-(-8))+f(-8)=\frac{1}{12}(x+8)-2=\frac{1}{12}x-\frac{4}{3}$$

2. (10%)

$$y=\frac{2\sqrt{x}}{3(1+\sqrt{x})}=\frac{2x^{1/2}}{3(1+x^{1/2})}$$

$$\Rightarrow dy=\left(\frac{x^{-1/2}(3(1+x^{1/2}))-2x^{1/2}(\frac{3}{2}x^{-1/2})}{9(1+x^{1/2})^2}\right)dx=\left(\frac{3x^{-1/2}+3-3}{9(1+x^{1/2})^2}\right)dx$$

$$\Rightarrow dy=\frac{1}{3\sqrt{x}(1+\sqrt{x})^2}dx$$

3.

(a) (10%)

An interior point(2 分) of the domain of a function f
where f' is zero(4 分) or undefined(4 分) is a critical point of f.

(b) (10%)

Suppose that $y=f(x)$ is continuous(3 分) on a closed interval $[a,b]$
and differentiable(3 分) on the interval's interior (a,b).
Then there is at least one point c in (a,b) at which

$\frac{f(b)-f(a)}{b-a}=f'(c)$. (4 分)

(c) (10%)

A point where the graph of a function has a tangent line(5 分) and
where the concavity changes(5 分) is a point of inflection.

4. (30%)

$$y = \frac{8x}{x^2+4} \;, y' = \frac{-8(x^2-4)}{(x^2+4)^2} \;, y'' = \frac{16x(x^2-12)}{(x^2+4)^3}$$

① The curve is **falling** on $(-\infty,-2)$ and $(2,\infty)$, and is **rising** on $(-2,2)$.

② Local and absolute **minimum** at $x=-2$.

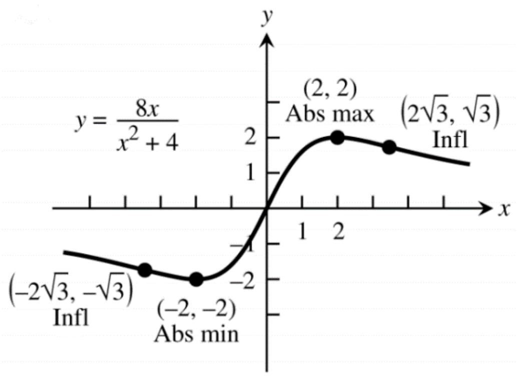
③ Local and absolute **maximum** at $x=2$.

④ The curve is **concave down** on $(-\infty,-2\sqrt{3})$ and $(0,2\sqrt{3})$.

⑤ The curve is **concave up** on $(-2\sqrt{3},0)$ and $(2\sqrt{3},\infty)$.

⑥ Point of **inflection** at $x=-2\sqrt{3}$, $x=0$, and $x=2\sqrt{3}$.

⑦ Horizontal **asymptote** : $y=0$.



\# 圖要標點，少標一個扣 1 分；沒畫線扣 3 分；沒畫圖扣 7 分

\# 上述①~⑦中，少/錯一個扣 2 分

5. (20%)

Let $(x,y)=(x,\frac{4}{3}x)$ be the coordinates of the corner that intersects the line.

Then $base = 3-x$ and $height = y = \frac{4}{3}x$, thus the area of the rectangle is

given by $A=(3-x)\left(\frac{4}{3}x\right)=4x-\frac{4}{3}x^2,\quad 0\le x\le 3.$

$$A' = 4-\frac{8}{3}x,$$

$$A' = 0 \Rightarrow x = \frac{3}{2},$$

$$A'' = -\frac{4}{3} \Rightarrow A''\left(\frac{3}{2}\right) < 0 \;\Rightarrow\; \text{local maximum at the critical point.}$$

The $base = 3-\frac{3}{2}=\frac{3}{2}$ and the $height = \frac{4}{3}(\frac{3}{2}) = 2.$