2020/11/09 微積分期中考 §2.1~§3.7 滿分為 110 分

整體批改標準：說明不清楚、不完整扣 3 分，過程沒寫或寫錯扣該題分數一半，符號標錯扣 2 分。

1.(a) (5%)

Let $f(x)$ be defined on an open interval about x_0, expect possibly at x_0 itself. We say that the limit of $f(x)$ as x approaches x_0 is the number L, and we write $\lim_{x\to x_0} f(x) = L$ (1 分), if, for every number $\varepsilon > 0$, (1 分) there exists a corresponding number $\delta > 0$ such that for all x (1 分), $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$ (2 分)

1.(b) (10%)

step1. $|\sqrt{4-x} - 2| < \varepsilon \Rightarrow -\varepsilon < \sqrt{4-x} - 2 < \varepsilon \Rightarrow 2 - \varepsilon < \sqrt{4-x} < 2 + \varepsilon$

$\Rightarrow (2-\varepsilon)^2 < 4 - x < (2+\varepsilon)^2 \Rightarrow 4 - (2+\varepsilon)^2 < x < 4 - (2-\varepsilon)^2$

step2. $|x - 0| < \delta \Rightarrow -\delta < x < \delta$

Then $-\delta = 4 - (2+\varepsilon)^2 = -\varepsilon^2 - 4\varepsilon \Rightarrow \delta = \varepsilon^2 + 4\varepsilon$, or

$\delta = 4 - (2-\varepsilon)^2 = 4\varepsilon - \varepsilon^2$

Thus choose the smaller distance, $\delta = 4\varepsilon - \varepsilon^2$.

$\forall \varepsilon, \exists \delta = 4\varepsilon - \varepsilon^2, s.t. \lim_{x\to 0} \sqrt{4-x} = 2$

Step1. (4 分)，Step2. (4 分)，結論(2 分)

* 結論要寫完整

2.(a) (5%)

If f is continuous function on a closed interval [a,b] (1 分)

and if y_0 is any value between $f(a)$ and $f(b)$, (1 分)

then $y_0 = f(c)$ for some c in [a,b]. (3 分)

2.(b) (10%)

$f(x) = x(x-1)^2$ is continuous on (-∞, ∞) (3 分)

f(0)=0, f(2)=2, f is also continuous on [0,2]⊂(-∞, ∞), 0≤ $y_0 = 1 \le 2$ (3 分)

By I.V.T., f(c)=1, for some c in [0,2]. (4 分)

By I.V.T.(1 分)，結論的區間(1 分)

3. (10%)

As defined,

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} ax + 2b = 2b \ \text{ and } \ \lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} x^2 + 3a - b = 3a - b,$$

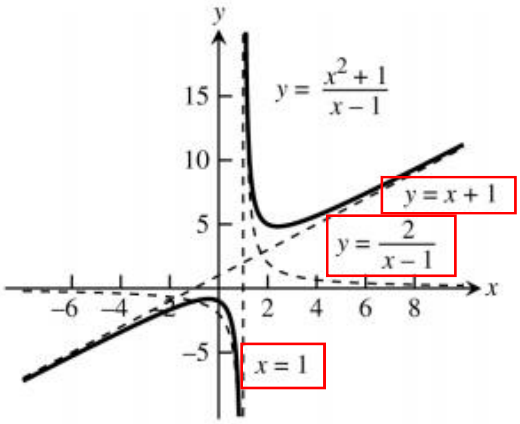
and

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} x^2 + 3a - b = 4 + 3a - b \ \text{ and } \ \lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} 3x - 5 = 1.$$

For g(x) to have 2b=3a−b and 4+3a−b=1 ⇒ a=b=$-\frac{3}{2}$

4. (10%)

$$y = \frac{x^2+1}{x-1} = x + 1 + \frac{2}{x-1}$$



(共 3 條，3 分/條)

5.(a) (5%)

If u and v are differentiable at x(1 分) and if $v(x) \neq 0$(1 分), then the quotient u/v is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

5.(b) (5%)

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\lim_{h\to 0}\frac{\frac{u(x+h)}{v(x+h)}-\frac{u(x)}{v(x)}}{h}=\lim_{h\to 0}\frac{v(x)u(x+h)-u(x)v(x+h)}{hv(x+h)v(x)}$$

$$=\lim_{h\to 0}\frac{v(x)u(x+h)-v(x)u(x)+v(x)u(x)-u(x)v(x+h)}{hv(x+h)v(x)}$$

$$=\lim_{h\to 0}\frac{v(x)\frac{u(x+h)-u(x)}{h}-u(x)\frac{v(x+h)-v(x)}{h}}{v(x+h)v(x)}$$ (2 分)

$$=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$ (1 分)

沒用 lim 扣 3 分

6.(a) (10%)

$$\lim_{x\to 1}\frac{x^{50}-1}{x-1}=50x^{49}\mid_{x=1}=50(1)^{49}=50$$

6.(b) (5%)

$$\lim_{x\to\infty}\frac{x+sinx+2\sqrt{x}}{x+sinx}=\lim_{x\to\infty}\frac{1+\frac{sinx}{x}+\frac{2}{\sqrt{x}}}{1+\frac{sinx}{x}}=\frac{1+0+0}{1+0}=1$$

7. (10%)

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathrm{f}(\theta)=2\left(\frac{\sin 3\theta}{1+\cos^2\theta}\right)\left(\frac{3\cos 3\theta(1+\cos^2\theta)-\sin 3\theta(-2\cos\theta\sin\theta)}{(1+\cos^2\theta)^2}\right)$$

$$=\frac{6\sin 3\theta\cos 3\theta+6\sin 3\theta\cos 3\theta\cos^2\theta+4\sin\theta\cos\theta(\sin 3\theta)^2}{(1+\cos^2\theta)^3}$$

8. (10%)

$$g(x)=\frac{1}{x^2}-1\ \Rightarrow\ g'(x)=-\frac{2}{x^3}\ \Rightarrow\ g(-1)=0\ and\ g'(-1)=2$$

$$f(u)=\left(\frac{3u-1}{5u+1}\right)^2\Rightarrow f'(u)=\frac{16(3u-1)}{(5u+1)^3}\Rightarrow f'\big(g(-1)\big)=f'(0)=-16$$

Therefore, $(f\circ g)'(-1)=f'(g(-1))g'(-1)=(-16)(2)=-32$ (4 分)

$g(x)$ (3 分)，$f(u)$ (3 分)

9.(a) (10%)

$$\cos(2x+3y)+x[-\sin(2x+3y)]\left(2+3\frac{dy}{dx}\right)=\frac{dy}{dx}\sin5x+5y\cos5x$$

$$\Rightarrow\ -(3x\sin(2x+3y)+\sin5x)\frac{dy}{dx}=5y\cos5x-\cos(2x+3y)+2x\sin(2x+3y)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{-5y\cos5x+\cos(2x+3y)-2x\sin(2x+3y)}{(3x\sin(2x+3y)+\sin5x)}$$

9.(b) (5%)

$$(x^2+y^2)^2=(x-y)^2$$

$$\Rightarrow 2(x^2+y^2)\left(2x+2y\,\frac{dy}{dx}\right)=2(x-y)\left(1-\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}[2y(x^2+y^2)+(x-y)]=-2x(x^2+y^2)+(x-y)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{-2x(x^2+y^2)+(x-y)}{2y(x^2+y^2)+(x-y)}$$