微 積 分

THOMAS' CALCULUS 12/E

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課程流水碼： U1130
開課班級： 通訊 1/電機 1/電資院 1
教學網站： http://www.hmwu.idv.tw

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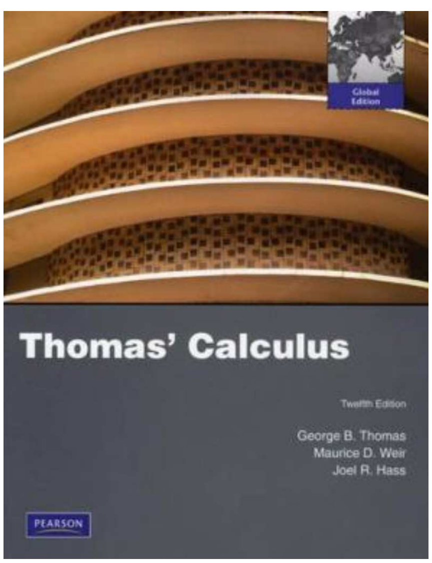

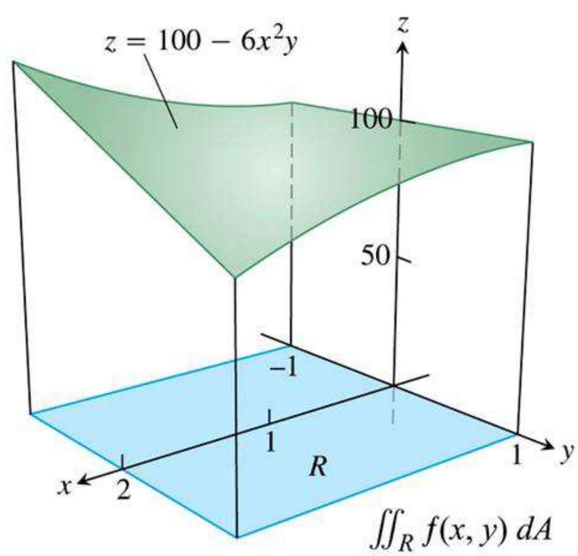

108 學年度第 2 學期

目錄

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叮嚀

A. 教不完是正常; 考不好是日常; 平常就要唸書!

B. 考過的題目，要主動訂正。

C. 上課以「互相尊重」為最高原則並盡到「告知老師」的義務。

D. 上課可小聲討論、可上廁所安靜去回、可飲食。(但請一定要維護教室整潔)

E. 四不一要: 「上課不聊天，睡覺不趴著，手機不要滑，考試不作弊，要認真。」

THOMAS' CALCULUS (12/E)

7.1 Inverse Function and Their Derivatives

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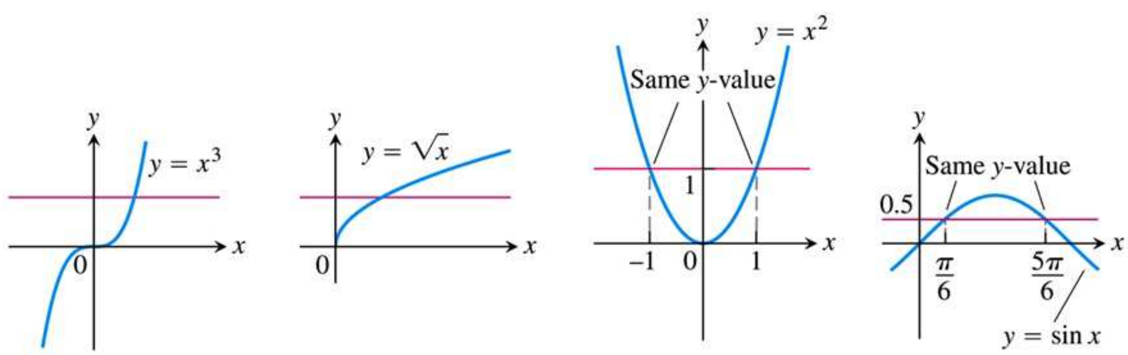
1 One-to-One Functions and Inverse Functions

1.1 *Definitions: One-to-One Function*

> A function $f(x)$ is one-to-one on a domain D if ________________ whenever ____________ in D.

1.2 One-to-one: (a) $y = x^3$, (b) $y = \sqrt{x}$. (圖示如下)

Not one-to-one: (c) $y = x^2$, (d) $y = \sin x$. (圖示如下)



1.3 A function $y = f(x)$ is one-to-one if and only if its graph interests each __________________ at most _________.

1.4 *Definitions: Inverse Function*

> Suppose that f is a one-to-one function on a domain D with range R. The inverse function ________ is defined by
>
> ______________ if ____________
>
> The ____________ of f^{-1} is ______ and the __________ of f^{-1} is ______.

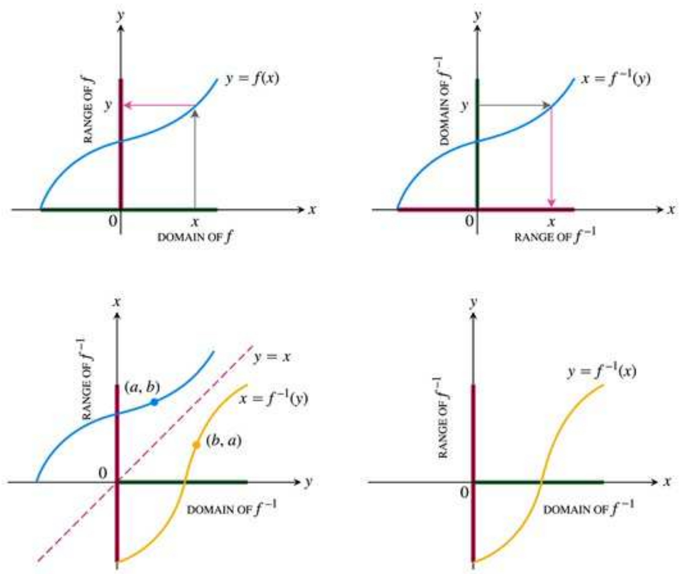
1.5 $(f^{-1} \cdot f)(x) =$ ______ for all x in the domain of f.

1.6 $(f \cdot f^{-1})(y) =$ ______ for all y in the domain of f^{-1}.

1.7 Only a one-to-one function can have an __________.

2 Finding Inverses

2.1 Determining the graph of $y = f^{-1}(x)$ from the graph of $y = f(x)$. (圖示如下)



2.2 Pass from f to f^{-1}.

(a) Solve the equation ____________ for x. This gives a formula ______________ where x is expressed as a function of y.

(b) Interchange ___________, obtaining a formula ______________ where f^{-1} is expressed in the conventional format with x as the _______________ variable and y as the _____________________.

Ex. 1 .... (`example3, p364`)

Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x.

sol:

✎ Ex. **2** .... (example4, p364)

Find the inverse of the function $y = x^2,\ x \geq 0$, expressed as a function of x.

sol:

3 Derivatives of Inverses of Differentiable Functions

3.1 $f(x) = (1/2)x + 1$ and $f^{-1}(x) =$__________.

$$\frac{d}{dx}f(x) = \underline{\hspace{5cm}}$$

$$\frac{d}{dx}f^{-1}(x) = \underline{\hspace{5cm}}$$

3.2 *Theorem 1: The Derivative Rule for Inverses*

(a) If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is ________________ at every point in its domain.

(b) The value of $(f^{-1})'$ at a point b in the domain of f^{-1} is the ______________ of f' the value of at the point $a = f^{-1}(b)$:

$$(f^{-1})'(b) = \underline{\hspace{4cm}} \quad \text{or} \quad \left.\frac{d}{dx}f^{-1}\right|_{x=b} = \underline{\hspace{4cm}}$$

3.3 When $y = f(x)$ is differentiable at $x = a$ and we change x by a small amount dx, the corresponding change in y is approximately ________________. This means that y changes about __________ times as fast as x when $x = a$ and that x changes about ____________ times as fast as y when $y = b$.

3.4 It is reasonable that the derivative of f^{-1} at b is the ______________ of the derivative of f at a.

✎ Ex. **3** .... (example5, p366)

Apply The Derivative Rule for Inverse Theorem to the function $f(x) = x^2$, $x \geq 0$.

sol:

✎ Ex. 4 .... (example6, p366)

Let $f(x) = x^3 - 2$. Find the value of df^{-1}/dx at $x = 6 = f(2)$ without finding a formula for $f^{-1}(x)$.

sol:

實習課練習 (EXERCISE 7.1)

21. Let $f(x) = x^3 - 1$. Find a formula for f^{-1}.

22. Let $f(x) = x^2 - 2x + 1,\ x \geq 1$. Find a formula for f^{-1}.

33. Let $f(x) = x^2 - 2x,\ x \leq 1$. Find f^{-1} and identify the domain and range of f^{-1}.

37. Let $f(x) = 5 - 4x,\ a = 1/2$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

38. Let $f(x) = 2x^2,\ x \geq 0,\ a = 5$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

41. Let $f(x) = x^3 - 3x^2 - 1,\ x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1 = f(3)$.

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7.2 Natural Logarithms

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1 Definition of the Natural Logarithm Function

1.1 The natural logarithm of a positive number x, written as ________, is the value of an integral.

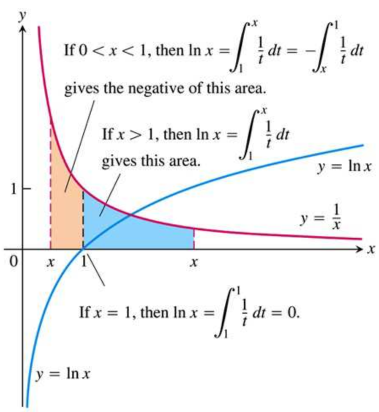
1.2 *Definitions: The Natural Logarithm Function*

$$\ln x = \underline{\qquad\qquad\qquad}, \quad x > 0$$

1.3 If $x > 1$, then $\ln x$ is the area under the curve ________ from ________ to ________.

1.4 For $0 < x < 1$, $\ln x$ gives the ________________ under the curve from ______ to ______.

1.5 The graph of $y = \ln x$ and its relation to the function $y = 1/x,\ x > 0$.
The graph of the logarithm rises above the x-axis as x moves from 1 to the right, and it falls below the axis as x moves from 1 to the left. (圖示如下)



1.6 *Definitions: The Number e*

> The number e is that number in the domain of the natural logarithm satisfying ____________.

1.7 $\ln 1 =$ _______________.

1.8 Geometrically, the number e corresponds to the point on the x-axis for which the area under the graph of __________ and above the interval ________ is the exact area of the unit square.

2 The Derivative of $y = \ln x$

2.1 $\frac{d}{dx} \ln x =$ ______________ $=$ ______

2.2 $y = \ln u$

$$\frac{d}{dx} \ln u = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad}, \quad u > 0$$

✎ Ex. 1 .... (example1, p371)

1. $\frac{d}{dx} \ln 2x =$
2. $\frac{d}{dx} \ln(x^2 + 3) =$
3. $\frac{d}{dx} \ln ax =$

3 Properties of Logarithms

3.1 *Theorem 2: Properties of Logarithms*

> For any numbers $a > 0$ and $x > 0$, the natural logarithm satisfies the following rules:
>
> (a) Product Rule: ____________________
>
> (b) Quotient Rule: ____________________
>
> (c) Reciprocal Rule: ________________
>
> (d) Power Rule: ________________

Ex. **2** .... (example2, p372)

1. $\ln 4 + \ln \sin x =$
2. $\ln \dfrac{x+1}{2x-3} =$
3. $\ln \sec x =$
4. $\ln \sqrt[3]{x+1} =$

4 The Integral $\int (1/u) du$

4.1 If u is a differentiable function that is never zero,

$$\int \frac{1}{u}\, du = \underline{\hspace{3cm}}.$$

4.2 If $u = f(x)$, then $du =$__________ and

$$\int \frac{1}{u}\, du = \frac{\hspace{3cm}}{\underline{\hspace{3cm}}} = \underline{\hspace{3.5cm}}.$$

Ex. **3** .... (example3, p374)

$$\int_0^2 \frac{2x}{x^2-5}\, dx$$

sol:

5 The Integral of $\tan x$, $\cot x$, $\sec x$ and $\csc x$

5.1 $\int \tan u\, du =$ ______________

Proof:

5.2 $\int \cot u \; du =$ ______________

Proof:

5.3 $\int \sec u \; du =$ ____________________

Proof:

5.4 $\int \csc u \; du =$ ______________________

Proof:

✐ Ex. 4 .... (example4, p375)

$$\int_0^{\pi/6} \tan 2x \; dx$$

sol:

6 Logarithmic Differentiation

✎ Ex. **5** .... (example5, p375)

Find dy/dx if $y = \dfrac{(x^2+1)(x+3)^{1/2}}{x-1}, \quad x > 1.$

sol:

實習課練習 (EXERCISE 7.2)

2. Express the following logarithms in terms of $\ln 5$ and $\ln 7$. (a) $\ln(1/125)$, (b) $\ln 9.8$, (c) $\ln 7\sqrt{7}$, (d) $\ln 1225$, (e) $\ln 0.056$, (f) $(\ln 35 + \ln(1/7))/(\ln 25)$.

3. Simplify the expressions: (a) $\ln\sin\theta - \ln(\frac{\sin\theta}{5})$, (b) $\ln(3x^2 - 9x) + \ln(\frac{1}{3x})$, (c) $\frac{1}{2}\ln(4t^4) - \ln 2$.

☐ Find the derivative of y with respect to x, or t, as appropriate.

8. $y = \ln(t^3/2)$

15. $y = t(\ln t)^2$

22. $y = \dfrac{x\ln x}{1 + \ln x}$

35. $y = \displaystyle\int_{x^2/2}^{x^2} \ln\sqrt{t}\, dt$

☐ Evaluate the integrals:

38. $\displaystyle\int_{-1}^{0} \frac{3}{3x-2}\, dx$

44. $\displaystyle\int_{2}^{4} \frac{dx}{x\ln x}$

45. $\displaystyle\int_{2}^{4} \frac{dx}{x(\ln x)^2}$

52. $\displaystyle\int_{0}^{\pi/12} 6\tan 3x\, dx$

53. $\displaystyle\int \frac{dx}{2\sqrt{x} + 2x}$

☐ Use logarithmic differentiaton to find the derivative of y with respect to the given independent variable.

55. $y = \sqrt{x(x+1)}$

62. $y = \dfrac{1}{t(t+1)(t+2)}$

67. $y = \sqrt[3]{\dfrac{x(x-2)}{x^2+1}}$

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7.3 Exponential Function

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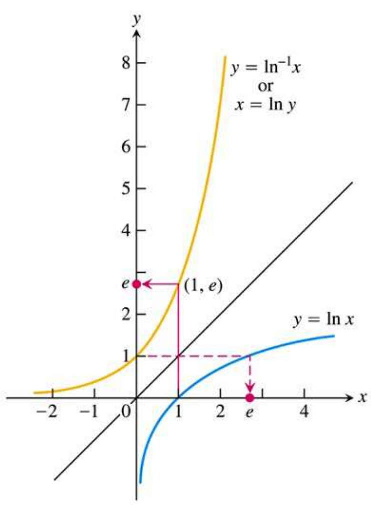
1 The Inverse of $\ln x$ and e^x

1.1 The function $\ln x$, being an increasing function of x with domain $(0, \infty)$ and range $(-\infty, \infty)$, has an inverse $\ln^{-1} x$ with domain $(-\infty, \infty)$ and range $(0, \infty)$. (圖示如下)



1.2 $\ln(e) =$______, $e =$ __________ = __________.

1.3 $e \doteq$ ____________________.

1.4 *Definitions: The Natural Exponential Function*

For every real number x, $e^x =$ __________ = __________.

1.5 $\ln e^r =$__________________ $\Rightarrow$ $e^r =$________________.

1.6 Inverse Equation for e^x and $\ln x$

$$e^{lnx} = \_\_\_\_\_\_, \quad \forall x > 0$$

$$\ln(e^x) = \_\_\_\_\_\_, \quad \forall x$$

Ex. 1 .... (example1, p378)

Solve the equation $e^{2x-6} = 4$ for x.

sol:

2 The Derivative and Integral of e^x

2.1 Let $f(x) = \ln x$ and $y = e^x =$ __________________. Then $\frac{dy}{dx} =$ _______

____

2.2 If u is any differentiable function of x, then $\frac{d}{dx}\, e^u =$ _________

2.3 $\int e^u \, du =$ __________

Ex. 2 .... (example2, p379)

(a) $\frac{d}{dx}\,(5e^x) =$

(b) $\frac{d}{dx}\, e^{-x} =$

(c) $\frac{d}{dx}\, e^{\sin x} =$

(d) $\frac{d}{dx}\, (e^{\sqrt{3x+1}}) =$

Ex. **3** .... (`example3, p379`)

(a) $\int_0^{\ln 2} e^{3x}\, dx =$

(b) $\int_0^{\pi/2} e^{\sin x} \cos x\, dx =$

3 Laws of Exponents

3.1 *Definitions: General Exponential Functions*

For any numbers $a > 0$ and x, the exponential function with base a is ____________.

3.2 *Theorem 3: Laws of Exponents for e^x*

For all numbers x, x_1 and x_2, the natural exponential e^x obeys the following laws:

(a) $e^{x_1} \cdot e^{x_2} =$__________

(b) $e^{-1} =$________

(c) $e^{x_1}/e^{x_2} =$__________

(d) $((e^{x_1})^{x_2}) =$________________

Proof of Law (a):

Ex. **4** .... (`example4, p382`) Differentiate $f(x) = x^x,\ x > 0$.

sol:

4 The Number e Expressed as a Limit

4.1 *Theorem 4: The Number e as a Limit*

The number e can be calculated as the limit: $e =$ ________________.

Proof:

4.2 *Power Rule (General Form)*

If u is a positive differentiable function of x and n is any real number, then u^n is a differentiable function of x and $\frac{d}{dx}u^n =$ ____________.

4.3 Examples:

(a) $\frac{d}{dx}x^{\sqrt{2}} =$

(b) $(2 + \sin 3x)^{\pi} =$

5 The Derivative of a^u

5.1 If $a > 0$, then $\frac{d}{dx}a^x =$__________.

Proof:

5.2 If $a > 0$ and u is a differentiable function of x, then a^u is a differentiable function of x and $\frac{d}{dx}a^u =$ ____________.

5.3 If $a \neq 1$, $\int a^u \ du =$ ____________.

Proof:

✎ Ex. **5** .... (example5, p383)

(a) $\frac{d}{dx} \ 3^x =$

(b) $\frac{d}{dx} \ 3^{-x} =$

(c) $\frac{d}{dx} \ 3^{\sin x} =$

(d) $\int 2^x \ dx =$

(e) $\int 2^{\sin x} \cos x \ dx =$

6 Logarithms with Base a

6.1 *Definitions:* $\log_a x$

For any positive number $a \neq 1$, _________ is the inverse function of a^x.

6.2 Inverse Equations for a^x and $log_a x$

$a^{\log_a x} =$______, $\quad x > 0$

$\log_a(a^x) =$______,$\forall x$

6.3 $\log_a x = \frac{}{}$

Proof:

6.4 Derivatives and Integrals Involving $\log_a x$:

$$\frac{d}{dx}(\log_a u) = \underline{\qquad\qquad}$$

Ex. **6** .... (example6, p385)

(a) $\dfrac{d}{dx}\log_{10}(3x+1) =$

(b) $\displaystyle\int \frac{\log_2 x}{x}\,dx =$

實習課練習 (EXERCISE 7.3)

☐ Solve for t.

2. (a) $e^{-0.01t} = 1000$, (b) $e^{kt} = \frac{1}{10}$, (c) $e^{(\ln 2)x} = 1/2$.

4. $e^{x^2} e^{2x+1} = e^t$

☐ Find the derivative of y with respect to x, t or θ, as appropriate.

9. $y = xe^x - e^x$

14. $y = \ln(3\theta e^{-\theta})$

20. $y = \ln(\dfrac{\sqrt{\theta}}{1+\sqrt{\theta}})$

24. $y = \displaystyle\int_{e^4\sqrt{x}}^{e^{2x}} \ln t \, dt$

25. $\ln y = e^y \sin x$

28. $\tan y = e^x + \ln x$

☐ Evaluate the integrals.

33. $\displaystyle\int 8e^{x+1} \, dx$

41. $\displaystyle\int \frac{e^{1/x}}{x^2} \, dx$

43. $\displaystyle\int_0^{\pi/4} (1 + e^{\tan\theta}) \sec^2\theta \, d\theta$

49. $\displaystyle\int \frac{e^r}{1+e^r} \, dr$

50. $\displaystyle\int \frac{dx}{1+e^x}$

☐ Find the derivative of y with respect to the given independent variable.

57. $y = 5^{\sqrt{s}}$

72. $y = \log_3 r \cdot \log_9 r$

81. $y = \log_2(8t^{\ln 2})$

☐ Evaluate the integrals.

85. $\displaystyle\int_0^1 2^{-\theta}\, d\theta$

91. $\displaystyle\int_2^4 x^{2x}(1+\ln x)\, dx$

96. $\displaystyle\int_1^e x^{(\ln 2-1)}\, dx$

99. $\displaystyle\int_1^4 \frac{\ln 2 \log_2 x}{x}\, dx$

106. $\displaystyle\int \frac{dx}{x(\log_8 x)^2}$

107. $\displaystyle\int_1^{\ln x} \frac{1}{t}\, dt, \quad x > 1$

☐ Find the derivative of y with respect to the given independent variable.

111. $y = x + 1^x$

116. $y = x^{\sin x}$

118. $y = (\ln x)^{\ln x}$

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7.5 Indeterminate Forms and L'Hopital's Rule

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1 Indeterminate Form 0/0

1.1 If the continuous functions $f(x)$ and $g(x)$ are both ________ at $x = a$, then ____________ cannot be found by substituting $x = a$.

1.2 The substitution produces ________, a meaningless expression, which we cannot evaluate. We use ________ as a notation for an expression known as an ____________________.

1.3 *Theorem: L'Hopital's Rule (First Form)*

> Suppose that ________________, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then
> $$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad}.$$

Proof:

1.4 *Theorem: L'Hopital's Rule (Stronger Form)*

> Suppose that ________________, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then
> $$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad},$$
> assuming that the limit on the right side exists.

1.5 *Using L'Hopital's Rule*

To find $\lim_{x \to a} \dfrac{f(x)}{g(x)}$ by L'Hopital's Rule,

(a) continue to differentiate f and g, so long as we still get the form ________ at $x = a$.

(b) But as soon as one or the other of these derivatives is different from ________ at $x = a$ we stop differentiating.

(c) L'Hopital's Rule does not apply when either the ______________ or ________________ has a finite ____________ limit.

Ex. 1 .... (`example1, p397`)

(a) $\lim_{x \to 0} \dfrac{3x - \sin x}{x} =$

(b) $\lim_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x} =$

(c) $\lim_{x \to 0} \dfrac{\sqrt{1+x} - 1 - x/2}{x^2} =$

(d) $\lim_{x \to 0} \dfrac{x - \sin x}{x^3} =$

Ex. 2 .... (example2, p398)

Find $\lim_{x\to 0} \frac{1-\cos x}{x+x^2}$.

sol:

Ex. 3 .... (example3, p398)

(a) $\lim_{x\to 0^+} \frac{\sin x}{x^2} =$

(b) $\lim_{x\to 0^-} \frac{\sin x}{x^2} =$

2 Indeterminate Form $\infty/\infty, \infty\cdot 0, \infty-\infty$

2.1 L'Hopital's Rule applies to the indeterminate form ________.

2.2 If ____________ and ____________ as $x \to a$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \_\_\_\_\_\_\_\_\_\_$$

provided the limit on the right exists.

2.3 In the notation $x \to a$ may be either ________ or _________.

2.4 Moreover $x \to a$ may be replaced by the one-sided limits _________ or _________.

Ex. 4 .... (example4, p398)

(a) $\lim_{x\to \pi/2} \frac{\sec x}{1+\tan x} =$

(b) $\lim_{x\to \infty} \frac{\ln x}{2\sqrt{x}} =$

(c) $\lim_{x\to\infty} \frac{e^x}{x^2} =$

Ex. 5 .... (example5, p399)

(a) Find $\lim_{x\to\infty}(x \sin\frac{1}{x})$.

(b) Find $\lim_{x\to 0^+}(\sqrt{x}\ln x)$.

sol:

Ex. 6 .... (example6, p399)

Find $\lim_{x\to 0}(\frac{1}{\sin x} - \frac{1}{x})$.

sol:

3 Indeterminate Powers

3.1 If $\lim_{x\to a} \ln f(x) = L$, then $\lim_{x\to a} f(x) =$ ________________ . Here a may be either finite or infinite.

3.2 *Theorem: Cauchy's Mean Value Theorem*

> Suppose functions f and g are continuous on $[a, b]$ and differentiable throughout (a, b) and also suppose $g'(x) \neq 0$ throughout (a, b). Then there exists a number c in (a, b) at which
>
> $$\frac{f'(c)}{g'(c)} = \underline{\qquad\qquad\qquad}.$$

Ex. 7 .... (example7, p400)

Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$

sol:

Ex. 8 .... (example8, p400)

Find $\lim\limits_{x\to\infty} x^{1/x}$.

sol:

實習課練習 (EXERCISE 7.5)

Use L'Hôpital Rule to find the limits.

5. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$.

8. $\lim_{x\to -5}\frac{x^2-25}{x+5}$.

14. $\lim_{t\to 0}\frac{\sin 5t}{2t}$.

20. $\lim_{x\to 1}\frac{x-1}{\ln x-\sin \pi x}$.

27. $\lim_{\theta\to 0}\frac{3^{\sin\theta}-1}{\theta}$.

29. $\lim_{x\to 0}\frac{x2^x}{2^x-1}$.

34. $\lim_{x\to 0^+}\frac{\ln(e^x-1)}{\ln x}$.

41. $\lim_{x\to 1^+}(\frac{1}{x-1}-\frac{1}{\ln x})$.

46. $\lim_{x\to\infty} x^2e^{-x}$.

48. $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$.

53. $\lim_{x\to\infty}(\ln x)^{1/x}$.

58. $\lim_{x\to 0}(e^x+x)^{1/x}$.

59. $\lim_{x\to 0^+} x^x$.

60. $\lim_{x\to 0^+}(1+\frac{1}{x})^x$.

62. $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

73. $\lim_{x\to\infty}\frac{e^{x^2}}{xe^x}$.

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7.6 Inverse Trigonometric Functions

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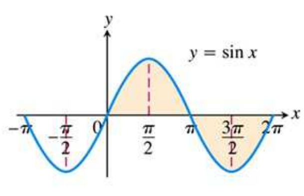
1 Defining the Inverses

1.1 Domain restrictions that make the trigonometric functions one-to-one.

(a) $y = \sin^{-1} x$ is the number in ____________ for which ____________.

(b) $y = \cos^{-1} x$ is the number in ________ for which ____________.

<table>
<tr><th>$\sin x$</th><th>$\sin^{-1} x$ or arcsin$x$</th></tr>
<tr><td>D: ____________
R: ________

</td><td>D: ________
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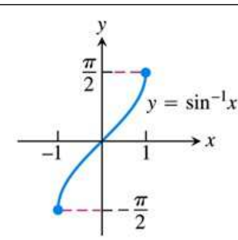
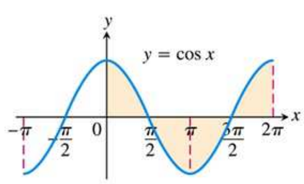
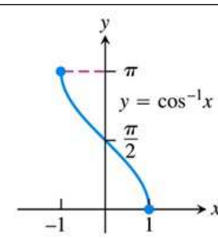
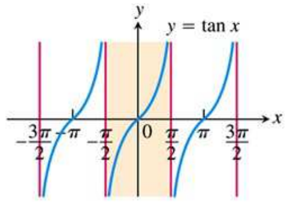
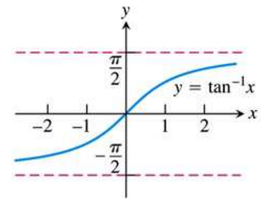
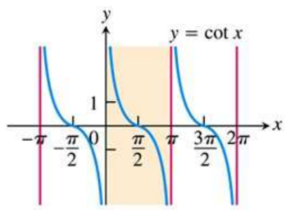
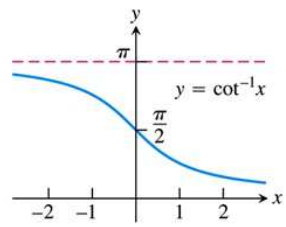
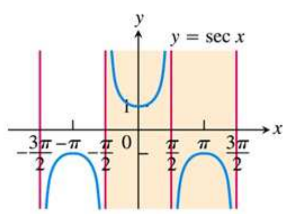
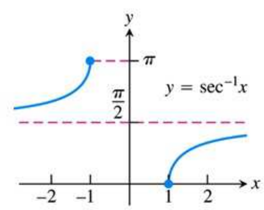
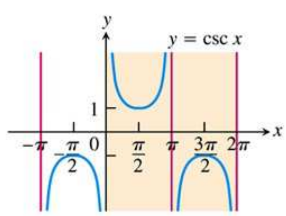
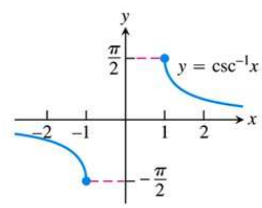

</td></tr>
</table>

<table>
<tr><th>$\cos x$</th><th>$\cos^{-1} x$ or arccos$x$</th></tr>
<tr><td>D: _______
R: ________

</td><td>D: ________
R: _______

</td></tr>
</table>

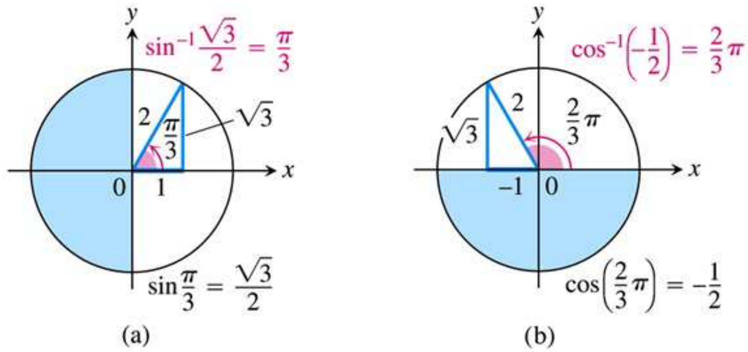
$\tan x$	$\tan^{-1} x$ or $\arctan x$
D: ______ R: ______  	D: ______ R: ______ 

$\cot x$	$\cot^{-1} x$ or $\operatorname{arccot} x$
D: ______ R: ______  	D: ______ R: ______ 

$\sec x$	$\sec^{-1} x$ or $\operatorname{arcsec} x$
D: ______ R: ______  	D: ______ R: ______ 

$\csc x$	$\csc^{-1} x$ or $\operatorname{arccsc} x$
D: ______ R: ______  	D: ______ R: ______ 

1.2 Common values of $\sin^{-1} x$:



x	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$
$\sin^{-1} x$						
$\cos^{-1} x$						

TABLE 1.3 Values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for selected values of θ

Degrees	−180	−135	−90	−45	0	30	45	60	90	120	135	150	180	270	360
θ (radians)	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{-\sqrt{2}}{2}$	−1	$\frac{-\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0
$\cos\theta$	−1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	−1	0	1
$\tan\theta$	0	1		−1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	−1	$\frac{-\sqrt{3}}{3}$	0		0

2 The Drivative of Inverse Trigonometric Functions

2.1 $(f^{-1}(x))' =$ ________________ .

2.2 Find the derivative of $y = \sin^{-1} x$.

2.3 Find the derivative of $y = \tan^{-1} x$.

2.4 Find the derivative of $y = \sec^{-1} x$.

2.5 $u = u(x)$

(a) $\frac{d}{dx}(\sin^{-1} u) =$ ________________ , $|u| < 1$.

(b) $\frac{d}{dx}(\tan^{-1} u) =$ ______________ .

(c) $\frac{d}{dx}(\sec^{-1} u) =$ ____________________________ .

2.6 *Inverse Function-Inverse Cofunction Identities*

(a) $\cos^{-1} x =$ ________________ $\Rightarrow \frac{d}{dx}(\cos^{-1} u) =$ __________________ , $|u| < 1$

(b) $\cot^{-1} x =$ ________________ $\Rightarrow \frac{d}{dx}(\cot^{-1} u) =$ ________________

(c) $\csc^{-1} x =$ ________________ $\Rightarrow \frac{d}{dx}(\csc^{-1} u) =$ ____________________ , $|u| > 1$

✎ Ex. **1** .... (`example4, p408`)

$\frac{d}{dx}(\sin^{-1} x^2) =$

sol:

✎ Ex. **2** .... (`example5, p410`)

$\frac{d}{dx}(\sec^{-1} 5x^4) =$

sol:

3 Integration Formulas

3.1 $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} =$ ________________, $u^2 < a^2$

3.2 $\displaystyle\int \frac{du}{a^2+u^2} =$ ________________

3.3 $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} =$ ________________, $|u| > a > 0$

✎ Ex. **3** .... (`example6, p412`)

(a) $\displaystyle\int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}} =$

(b) $\displaystyle\int \frac{dx}{\sqrt{3-4x^2}} =$

(c) $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-6}} =$

✎ Ex. **4** .... (example7(a), p412)

Evaluate $\int \frac{dx}{\sqrt{4x - x^2}}$

sol:

✎ Ex. **5** .... (example7(b), p412)

Evaluate $\int \frac{dx}{4x^2 + 4x + 2}$

sol:

實習課練習 (EXERCISE 7.6)

☐ Find the values.

9. $\sin(\cos^{-1}(\frac{\sqrt{2}}{2}))$

12. $\cot(\sin^{-1}(-\frac{\sqrt{3}}{2}))$

☐ Find the derivative of y with respect to the appropriate variable.

22. $y = \cos^{-1}(1/x)$ $y = \sin^{-1}(3/t^2)$

33. $y = \ln(\tan^{-1} x)$

41. $y = x\sin^{-1} x + \sqrt{1-x^2}$

42. $y = \ln(x^2+4) - x\tan^{-1}(\frac{x}{2})$

☐ Evaluate the integrals.

44. $\displaystyle\int \frac{dx}{\sqrt{1-4x^2}}$

46. $\displaystyle\int \frac{dx}{9+3x^2}$

47. $\displaystyle\int \frac{dx}{x\sqrt{25x^2-2}}$

63. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$

68. $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$

72. $\displaystyle\int \frac{dy}{y^2+6y+10}$

79. $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$

83. $\displaystyle\int \frac{(\sin^{-1} x)^2\,dx}{\sqrt{1-x^2}}$

86. $\displaystyle\int \frac{dy}{(\sin^{-1} y)\sqrt{1-y^2}}$

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8.1 Integration by Parts

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1 Basic Integration Formulas

1.1 Substitution Rule: $u = g(x)$, ____________________

1.2 Basic formulas:

(1) $\int du =$ ________

(2) $\int k\ du =$ ________

(3) $\int (du + dv) =$ ________

(4) $\int u^n\ du =$ ________, $\quad n \neq -1$

(5) $\int \frac{du}{u} =$ ________

(6) $\int \sin u\ du =$ ________

(7) $\int \sec^2 u\ du =$ ________

(8) $\int \csc^2 u\ du =$ ________

(9) $\int \sec u \tan u\ du =$ ________

(10) $\int \csc u \cot u\ du =$ ________

(11) $\int \tan u\ du =$ ________ $=$ ________

(12) $\int \cot u \; du =$ ______________ = ________________

(13) $\int e^u \; du =$ _________

(14) $\int a^u \; du =$ __________

(15) $\int \frac{du}{\sqrt{a^2 - u^2}} =$ ______________

(16) $\int \frac{du}{a^2 + u^2} =$ ________________

(17) $\int \frac{du}{u\sqrt{u^2 - a^2}} =$ ________________

2 Product Rule in Integral Form

2.1 Integration by parts: if f and g are differentiable functions of x,

(a) The Product Rule: $\frac{d}{dx}[f(x)g(x)] =$ ______________________.

(b) In term of indefinite integrals:

$$\int \frac{d}{dx}[f(x)g(x)] \; dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

(c) Integration by parts:

____.

2.2 Let $u = f(x)$ and $v = g(x)$, then $du = f'(x) \; dx$ and $dv = g'(x) \; dx$. Using substitution rule, the integration by parts formula becomes _______________________.

2.3 Note:

(a) The goal of integration by parts is to go from an integral __________ that we don't see how to evaluate to an integral __________ that we can evaluate.

(b) The integration by parts does not always work.

2.4 Integration by Parts Formula for Definite Integrals

$$\int_a^b f(x)g'(x) \; dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Ex. 1 .... (example1, p437)

Find $\int x \cos x \; dx$

sol:

Ex. 2 .... (example2, p3)563

Find $\int \ln x \; dx$

sol:

Ex. 3 .... (example3, p437)

Find $\int x^2 e^x \; dx$

sol:

Ex. 4 .... (example4, p438)

Find $\int e^x \cos x \; dx$

sol:

3 Tabular Integration

✎ Ex. **5** .... (example7, p440)

Evaluate $\int x^2 e^x \; dx$.

sol:

✎ Ex. **6** .... (`example8, p441`)

Evaluate $\int x^3 \sin x \; dx$

sol:

實習課練習 (EXERCISE 8.1)

1. $\int x \sin \frac{x}{2}\ dx$

6. $\int_1^e x^3 \ln x\ dx$

16. $\int p^4 e^{-p}\ dp$

19. $\int x^5 e^x\ dx$

21. $\int e^\theta \sin\theta\ d\theta$

25. $\int e^{\sqrt{3s+9}}\ ds$

28. $\int \ln(x+x^2)\ dx$

36. $\int \frac{(\ln x)^3}{x}\ dx$

44. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

46. $\int \sqrt{x} e^{\sqrt{x}}\ dx$

62. Establish the reduction formula: $\int x^n \sin x\ dx = -x^n \cos x + n \int x^{n-1} \cos x\ dx.$

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8.2 Trigonometric Integrals

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1 Products of Powers of Sines and Cosines

1.1 $\int \sin^m x \cos^n x \, dx$

Case 1. If m is ________ $\Rightarrow m =$ __________, use ____________________

$$\sin^m x = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Case 2. If m is ________ and n is ________. $\Rightarrow n =$ __________, use ____________________

$$\cos^n x = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Case 3. If both m and n are ________ $\Rightarrow$ use $\sin^2 x =$ ____________, $\cos^2 x =$ ____________.

✎ **Ex. 1** .... (example1, p444)

Evaluate $\int \sin^3 x \cos^2 x \, dx$.

sol:

✎ Ex. 2 .... (example2, p445)

Evaluate $\int \cos^5 x \; dx$.

sol:

✎ Ex. 3 .... (example3, p445)

Evaluate $\int \sin^2 x \cos^4 x \; dx$.

sol:

2 Eliminating Square Roots

2.1 Use $\cos^2 \theta =$ ____________

✎ Ex. 4 .... (example4, p446)

Evaluate $\int_0^{\pi/4} \sqrt{1 + \cos 4x} \; dx$.

sol:

3 Integrals of Powers of $\tan x$ and $\sec x$

3.1 Use $\tan^2 x =$ ____________

3.2 Use $\sec^2 x =$ ____________

✎ Ex. 5 .... (example5, p446)

Evaluate $\int \tan^4 x \; dx$.

sol:

✎ Ex. 6 .... (example6, p447)

Evaluate $\int \sec^3 x \; dx$.

sol:

4 Products of Sine and Cosines

4.1 $\sin mx \sin nx =$ ______________________________.

4.2 $\sin mx \cos nx =$ ______________________________.

4.3 $\cos mx \cos nx =$ ______________________________.

✎ Ex. **7** .... (`example7, p448`)

Evaluate $\int \sin 3x \cos 5x \; dx$.

sol:

實習課練習 (EXERCISE 8.2)

8. $\displaystyle\int_0^{\pi} \sin^5 \frac{x}{2}\ dx$

16. $\displaystyle\int 7\cos^7 t\ dt$

17. $\displaystyle\int_0^{\pi} 8\sin^4 x\ dx$

22. $\displaystyle\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\ d\theta$

23. $\displaystyle\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\ dx$

25. $\displaystyle\int_0^{\pi} \sqrt{1-\sin^2 t}\ dt$

31. $\displaystyle\int_0^{\pi/2} \theta\sqrt{1-\cos 2\theta}\ d\theta$

35. $\displaystyle\int \sec^3 x \tan x\ dx$

38. $\displaystyle\int \sec^4 x \tan^2 x\ dx$

53. $\displaystyle\int_{-\pi}^{\pi} \sin 3x \sin 3x\ dx$

55. $\displaystyle\int \cos 3x \cos 4x\ dx$

57. $\displaystyle\int \sin^2 \theta \cos 3\theta\ d\theta$

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8.3 Trigonometric Substitutions

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1 Three Basic Substitutions

1.1 Integrals involving $\sqrt{a^2 - x^2}$:

Let $x =$ __________,

$a^2 - x^2 =$ __________ $=$ __________ $=$ __________

1.2 Integrals involving $\sqrt{a^2 + x^2}$:

Let $x =$ __________,

$a^2 + x^2 =$ __________ $=$ __________ $=$ __________

1.3 Integrals involving $\sqrt{x^2 - a^2}$:

Let $x =$ __________,

$x^2 - a^2 =$ __________ $=$ __________ $=$ __________

1.4 (a) $\sqrt{a^2 + x^2} =$ __________

(b) $\sqrt{a^2 - x^2} =$ __________

(c) $\sqrt{x^2 - a^2} =$ __________

圖示如下:

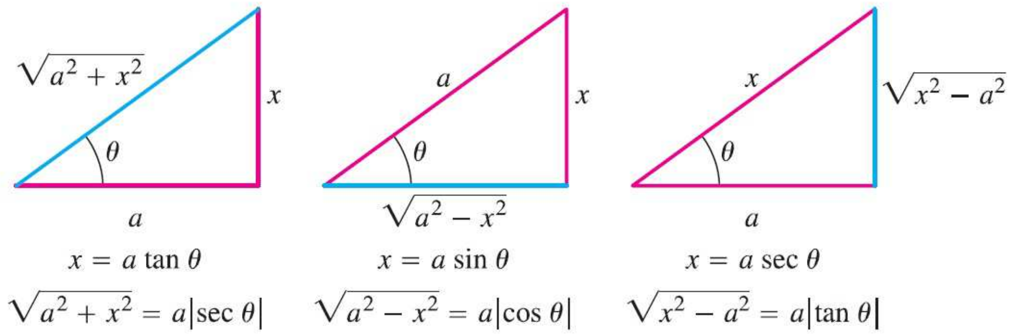

FIGURE 8.2 Reference triangles for the three basic substitutions identifying the sides labeled x and a for each substitution.

✎ Ex. **1** .... (example1, p450)

Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$.

sol:

✎ Ex. **2** .... (example2, p451)

Evaluate $\int \frac{x^2\ dx}{\sqrt{9-x^2}}$

sol:

✐ Ex. **3** .... (example3, p451)

Evaluate $\int \frac{dx}{\sqrt{25x^2-4}}, \quad x > 2/5.$

sol:

實習課練習 (EXERCISE 8.3)

4. $\displaystyle\int_0^2 \frac{dx}{8+2x^2}$

7. $\displaystyle\int \sqrt{25-t^2}\; dt$

10. $\displaystyle\int \frac{5\; dx}{\sqrt{25x^2-9}}, \quad x > 3/5$

17. $\displaystyle\int \frac{x^3\; dx}{\sqrt{x^2+4}}$

20. $\displaystyle\int \frac{\sqrt{9-w^2}}{w^2}\; dw$

33. $\displaystyle\int \frac{v^2}{(1-v^2)^{5/2}}\; dv$

43. $\displaystyle\int \frac{x\; dx}{\sqrt{1+x^4}}$

46. $\displaystyle\int \sqrt{\frac{x}{1-x^3}}\; dx$

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8.4 Integration of Rational Functions by Partial Fractions

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1 Partial Fractions

1.1 This section shows how to express a rational function (a quotient of polynomials) as a sum of simpler fractions, called ________________,

Example: $\dfrac{5x-3}{x^2-2x-3} =$ ________________

$\displaystyle\int \frac{5x-3}{x^2-2x-3} =$ ____________________ = ______________________________

1.2 The method of partial fractions: Example: $\dfrac{5x-3}{x^2-2x-3} =$ ________________

Ex. 1 .... (`example1, p455`)

Use partial fractions to evaluate $\displaystyle\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$ (Distinct linear factors)

sol:

Ex. **2** . . . . (`example2, p455`)

Use partial fractions to evaluate $\int \frac{6x+7}{(x+2)^2}\,dx$ (A repeated linear factor)

sol:

Ex. **3** . . . . (`example3, p456`)

Use partial fractions to evaluate $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$ (An improper fraction)

sol:

Ex. **4** . . . . (`example4, p456`)

Use partial fractions to evaluate $\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$ (An irreducible quadratic factor)

sol:

✎ Ex. 5 .... (example5, p457)

Use partial fractions to evaluate $\int \frac{dx}{x(x^2+1)^2}\, dx$ (An repeated irreducible quadratic factor)

sol:

2 The Heaviside Method for Linear Factors

2.1 *Heaviside Method*

Write the partial-fraction expansion of $f(x)/g(x)$ as

$$\frac{f(x)}{g(x)} = \underline{\hspace{6cm}},$$

where

$$A_1 = \underline{\hspace{4cm}}$$

$$A_2 = \underline{\hspace{5cm}}$$

$$\vdots$$

$$A_n = \underline{\hspace{6cm}}$$

✎ Ex. 6 .... (example6, p458)

Find A, B and C in the equation: $\frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

sol:

✎ Ex. **7** .... (example7, p459)

Use the Heaviside Method to evaluate $\displaystyle\int \frac{x+4}{x^3+3x^2-10x}\,dx$.

sol:

✎ Ex. **8** .... (example8, p460)

Find A, B and C in the equation: $\displaystyle\frac{x-1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

sol:

✎ Ex. **9** .... (`example9, p460`)

Find A, B and C in the expression: $\dfrac{x^2+1}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

sol:

實習課練習 (EXERCISE 8.4)

☐ Expand the quotients by partial fractions.

2. $\dfrac{5x-7}{x^2-3x+2}$

5. $\dfrac{z+1}{z^2(z-1)}$

8. $\dfrac{t^4+9}{t^4+9t^2}$

. . . .

☐ Express the integrands as a sum of partial fractions and evaluate the integrals.

16. $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx$

17. $\displaystyle\int_0^1 \frac{x^3\,dx}{x^2+2x+1}$

20. $\displaystyle\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$

24. $\displaystyle\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$

33. $\displaystyle\int \frac{2x^3-2x^2+1}{x^2-x}\,dx$

39. $\displaystyle\int \frac{e^t\,dt}{e^{2t}+3e^t+2}$

42. $\displaystyle\int \frac{\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$

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8.7 Improper Integrals

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1 Infinite Limits of Integration

1.1 *Definitions: Type I Improper Integrals*

Integrals with ______________ of integration are **improper integrals of Type I**.

(a) If $f(x)$ is continuous on $[a, \infty)$, then
$$\int_a^\infty f(x)\, dx = \underline{\hspace{5cm}}$$

(b) If $f(x)$ is continuous on $(-\infty, b]$, then
$$\int_{-\infty}^b f(x)\, dx = \underline{\hspace{5cm}}$$

(c) If $f(x)$ is continuous on $(-\infty, \infty)$ then
$$\int_{-\infty}^\infty f(x)\, dx = \underline{\hspace{8cm}}$$

where c is any real number. In each case, if the limit is ________ we say that the improper integral ____________ and that the ________ is the **value** of the improper integral. If the limit fails to exist, the improper integral ___________ (the area under the curve is infinite).

1.2 Examples:

(a) Upper limit: $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\, dx = \underline{\hspace{5cm}}$

(b) Lower limit: $\displaystyle\int_{-\infty}^0 \frac{dx}{1+x^2} = \underline{\hspace{5cm}}$

(c) Both limits: $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} =$ ______________________________.

✎ Ex. 1 .... (example1, p479)

Is the area under the curve $y = (\ln x)/x^2$ from $x = 1$ to $x = \infty$ finite? If so, what is it?

sol:

✎ Ex. 2 .... (example2, p479)

Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

sol:

✎ Ex. 3 .... (example2, p480)

For what values of p does the integral $\displaystyle\int_{1}^{\infty} \frac{1}{x^p}\, dx$ converge? When the integral does converge, what is its value?

sol:

2 Integrands with Vertical Asymptotes

2.1 *Definitions: Type II Improper Integrals*

Integrals of functions that become ________________ within the interval of integration are **improper integrals of Type II**.

(a) If $f(x)$ is continuous on $(a, b]$ and is discontinuous at a then

$$\int_a^b f(x)\,dx = \underline{\hspace{4cm}}$$

(b) If $f(x)$ is continuous on $[a, b)$ and is discontinuous at b, then

$$\int_a^b f(x)\,dx = \underline{\hspace{4cm}}$$

(c) If $f(x)$ is discontinuous at c, where $a < c < b$, and continuous on $[a, c) \cup (c, b]$ then

$$\int_a^b f(x)\,dx = \underline{\hspace{5cm}}$$

In each case, if the limit is finite we say the improper integral ____________ and that the limit is the value of the improper integral. If the limit does not exist, the integral ___________.

2.2 Examples:

(a) Upper endpoint: $\displaystyle\int_0^1 \frac{dx}{(x-1)^{2/3}} = \underline{\hspace{4cm}}$

(b) Lower endpoint: $\displaystyle\int_1^3 \frac{dx}{(x-1)^{2/3}} = \underline{\hspace{4cm}}$

(c) Interior point: $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}} = \underline{\hspace{6cm}}$

Ex. 4 .... (example4, p482)

Investigate the convergence of $\displaystyle\int_0^1 \frac{1}{1-x}\,dx$.

sol:

Ex. 5 .... (example5, p482)

Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

sol:

3 Tests for Convergence and Divergence

3.1 *Theorem 1: Direct Comparison Test*

Let f and g be continuous on $[a, \infty)$ with ________________ for all $x \geq a$. Then

(a) $\int_a^\infty f(x)\ dx$ ____________ if $\int_a^\infty g(x)\ dx$ ____________.

(b) $\int_a^\infty g(x)\ dx$ ___________ if $\int_a^\infty f(x)\ dx$ ___________.

3.2 *Theorem 2: Limit Comparison Test*

If the positive functions f and g are continuous on $[a, \infty)$ and if ________________, $0 < L < \infty$, then ______________ and ______________ both ____________ or both ___________.

Ex. 6 .... (example6, p483)

Does the integral $\int_1^\infty e^{-x^2}\ dx$ converges?

sol:

Ex. 7 .... (example7, p484)

(a) $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ converges because

(b) $\int_1^\infty \frac{1}{\sqrt{x^2-0.1}}\,dx$ diverges because

Ex. 8 .... (example8, p485)

Show that $\int_1^\infty \frac{dx}{1+x^2}$ converges and find the integral value.

sol:

Ex. 9 .... (example9, p485)

Investigate the convergence of $\int_1^\infty \frac{1-e^{-x}}{x}\,dx$.

sol:

實習課練習 (EXERCISE 8.7)

☐ Evaluate the integrals.

2. $\displaystyle\int_1^\infty \frac{dx}{x^{1.001}}$

7. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

10. $\displaystyle\int_{-\infty}^2 \frac{2\ dx}{x^2+4}$

13. $\displaystyle\int_{-\infty}^\infty \frac{2x\ dx}{(x^2+1)^2}$

17. $\displaystyle\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$

22. $\displaystyle\int_0^\infty 2e^{-\theta}\sin\theta\ d\theta$

25. $\displaystyle\int_0^1 x\ln x\ dx$

30. $\displaystyle\int_2^4 \frac{dt}{t\sqrt{t^2-4}}$

32. $\displaystyle\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

☐ Test the integrals for convergence.

39. $\displaystyle\int_0^{\ln x} x^{-2}e^{-1/x}dx$

45. $\displaystyle\int_{-1}^1 \ln|x|\ dx$

54. $\displaystyle\int_2^\infty \frac{x\ dx}{\sqrt{x^4-1}}$

58. $\displaystyle\int_2^\infty \frac{1}{\ln x}\ dx$

61. $\displaystyle\int_1^\infty \frac{1}{\sqrt{e^x-x}}$

THOMAS' CALCULUS (12/E)

10.1 Sequences

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1 Sequences, Convergence and Divergence

1.1 A sequence is ____________________(_______________________) in a given order.

1.2 Each of $a_1, a_2, \cdots$ are the __________ of the sequence.

1.3 Example: $2, 4, 6, 8, 10, 12, \cdots, 2n, \cdots$ has first term $a_1 = 2$, second term $a_2 = 4$ and _____________ (_____________). The integer n is called the __________ of a_n.

1.4 *Definitions: Infinite Sequence*

> An infinite sequence of numbers is a ____________ whose ____________ is the set of ____________________.

1.5 The sequence $1, 2, 3, 4, \cdots$ is not the same as the sequence $2, 1, 3, 4, \cdots$; __________ is important.

1.6 Examples:

nth term	listing terms	write
$a_n = \sqrt{n}$	$\{a_n\} =$ ____________________	$\{a_n\} =$ __________
$b_n = (-1)^{n+1}\frac{1}{n}$	____________________	____________________
$c_n = \frac{n-1}{n}$	____________________	____________________
$d_n = (-1)^{n+1}$	____________________	____________________

1.7 *Definitions: Converges, Diverges, Limit*

The sequence $\{a_n\}$ converges to the number ______ if to every positive number ______ there corresponds an integer _______ such that for all ______,

____

If no such number L exists, we say that $\{a_n\}$ ____________. If $\{a_n\}$ converges to L, we write __________________ or simply ____________ and call L the _________ of the sequence.

1.8 Examples:

(a) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots, \frac{1}{n}, \cdots\}$; $\lim_{n\to\infty} a_n =$ ______

(b) $\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots, 1-\frac{1}{n}, \cdots\}$; $\lim_{n\to\infty} a_n =$ ______

(c) $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \cdots, \sqrt{n}, \cdots\}$; ________________

(d) $\{1, -1, 1, -1, \cdots, (-1)^{n+1}, \cdots\}$; ______________

1.9 *Definitions: Diverges to Infinity*

The sequence $\{a_n\}$ ____________ to infinity if for every number _______ there is an integer _______ such that for all ______ larger than N, ____________. If this condition holds we write

____________________ or _____________

Similarly if for every number m there is an integer N such that for all $n > N$ we have $a_n < m$ then we say $\{a_n\}$ diverges to negative infinity and write

_____________________ or ______________

1.10 A sequence may diverge without diverging to infinity or negative infinity. Examples: _____________________ and ________________________________.

2 Calculating Limits of Sequences

2.1 *Theorem 1*

Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers and let A and B be real numbers. The following rules hold if $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$

1. Sum Rule: __________________________
2. Difference Rule: __________________________

3. Product Rule: ____________
4. Constant Multiple Rule: ____________
5. Quotient Rule: ____________

2.2 *Theorem 2: The Sandwich Theorem for Sequences*

Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers. If ____________ holds for all n beyond some index N, and if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n =$ ______ then ____________ also.

2.3 *Theorem 3: The Continuous Function Theorem for Sequences*

Let $\{a_n\}$ be a sequence of real numbers. If ________ and if f is a function that is ____________ and defined at all a_n, then ____________.

Ex. 1 .... (example3, p536)

(a) $\lim_{n\to\infty}(-\frac{1}{n}) =$

(b) $\lim_{n\to\infty}(\frac{n-1}{n}) =$

(c) $\lim_{n\to\infty}\frac{5}{n^2} =$

(d) $\lim_{n\to\infty}\frac{4-7n^6}{n^6+3} =$

Ex. 2 .... (example4, p536)

(a) $\lim_{n\to\infty}(\frac{\cos n}{n}) =$

(b) $\lim_{n\to\infty}(\frac{1}{2^n}) =$

(c) $\lim_{n\to\infty}(-1)^n\frac{1}{n} =$

Ex. 3 .... (example5, p537)

Show that $\sqrt{(n+1)/n} \to 1$.

sol:

Ex. 4 .... (example6, p537)

Find $\lim_{n\to\infty} 2^{1/n}$.

sol:

3 Using L'Hopital's Rule

3.1 *Theorem 4*

Suppose that $f(x)$ is a function defined for all $x \geq n_0$ and that is a sequence of real numbers such that ____________ for $n \geq n_0$. Then ______________ $\Rightarrow$ ______________

Ex. 5 .... (example7, p537)

Show that $\lim_{n\to\infty} \frac{\ln n}{n} = 0$.

sol:

Ex. 6 .... (`example8, p537`)

Does the sequence whose nth term is $a_n = (\frac{n+1}{n-1})^n$ converges? If so, find $\lim_{n\to\infty} a_n$.

sol:

4 Commonly Occurring Limits

4.1 *Theorem 5*

1. $\lim_{n\to\infty} \frac{\ln n}{n} =$ ______
2. $\lim_{n\to\infty} \sqrt[n]{n} =$ ______
3. $\lim_{n\to\infty} x^{1/n} =$ ______ $(x > 0)$
4. $\lim_{n\to\infty} x^n =$ ______ $(|x| < 1)$
5. $\lim_{n\to\infty} (1 + \frac{x}{n})^n =$ ______ (any x)
6. $\lim_{n\to\infty} \frac{x^n}{n!} =$ ______ (any x)

4.2 *Definition*

A sequence $\{a_n\}$ is ____________ if __________ for all n. That is, ______________. The sequence is ____________ if __________ for all n. The sequence $\{a_n\}$ is __________ if it is either nondecreasing or nonincreasing.

4.3 *Theorem 6: The Monotonic Sequence Theorem*

If a sequence $\{a_n\}$ is both _________ and __________, then the sequence __________.

Ex. 7 .... (example9, p538)

(a) $\lim\limits_{n\to\infty} \dfrac{\ln(n^2)}{n} =$

(b) $\lim\limits_{n\to\infty} \sqrt[n]{n^2} =$

(c) $\lim\limits_{n\to\infty} \sqrt[n]{3n} =$

(d) $\lim\limits_{n\to\infty} (-\dfrac{1}{2})^n =$

(e) $\lim\limits_{n\to\infty} (\dfrac{n-2}{n})^n =$

(f) $\lim\limits_{n\to\infty} \dfrac{100^n}{n!} =$

實習課練習 (EXERCISE 10.1)

☐ Find the values of a_1, a_2, a_3 and a_4.

3. $a_n = \dfrac{(-1)^{n+1}}{2n-1}$

☐ Write out the first ten terms of the sequence.

9. $a_1 = 2, a_{n+1} = (-1)^{n+1}a_n/2$

☐ Find a formula for the nth term of the sequence.

16. The sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \cdots$

19. The sequence $0, 3, 8, 15, 24, \cdots$

☐ Which of the sequence $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence.

27. $a_n = 2 + (0.1)^n$

31. $a_n = \dfrac{1-5n^4}{n^4+8n^3}$

36. $a_n = (-1)^n(1 - \frac{1}{n})$

43. $a_n = \sin(\frac{\pi}{2} + \frac{1}{n})$

54. $a_n = (1 - \frac{1}{n})^n$

57. $a_n = (\frac{3}{n})^{1/n}$

63. $a_n = \dfrac{n!}{n^n}$

71. $a_n = (\dfrac{x^n}{2n+1})^{1/n}, \quad x > 0$

84. $a_n = \sqrt[n]{n^2+n}$

86. $a_n = \dfrac{(\ln n)^5}{\sqrt{n}}$

89. $a_n = \dfrac{1}{n}\displaystyle\int_1^n \frac{1}{x}\,dx$

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10.2 Infinite Series

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1 Series

1.1 *Definitions: Infinite Series, nthTerm, Partial Sum, Converges, Sum*

(a) Given a sequence of numbers $\{a_n\}$, an expression of the form

____

is an ________________. The number a_n is the ____________ of the series.

(b) The sequence $\{s_n\}$ defined by

$$s_n = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

is the sequence of _______________ of the series, the number being the ________ partial sum.

(c) If _____________________________ converges to a limit L, we say that _____________________ and that its sum is ______. In this case, we also write

$$a_1 + a_2 + \cdots + a_n + \cdots = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(d) If the sequence of partial sums of the series does not converge, we say that the series ___________.

1.2 The series of the form

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_$$

is called ___________________, in which a and r are fixed real number and $a \neq 0$.

(a) Examples: $a = 1$

i. $r = \frac{1}{2}$,

ii. $r = -\frac{1}{3}$,

iii. $r = 1$

(b) $s_n =$

(c) If $|r| < 1$, the geometric series converges: ____________________, $|r| < 1$.

(d) If $|r| \geq 1$, the series ____________.

✎ Ex. **1** .... (example1, p546)

$$\sum_{n=1}^{\infty} \frac{1}{9}(\frac{1}{3})^{n-1} =$$

✎ Ex. **2** .... (example2, p546)

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5}{4^n} =$$

✎ Ex. **3** .... (example4, p547)

Express the repeating decimal 5.232323... as the ratio of two integers.

sol:

Ex. 4 .... (example5, p547)

Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

sol:

2 The nth-Term Test for Divergence

2.1 *Theorem 7*

If ____________ converges, then ___________.

2.2 *The nth-Term Test for Divergence*

$\sum_{n=1}^{\infty} a_n$ diverges if ______________ fails to _________ or is different from _________.

2.3 *Theorem 8*

If $\sum a_n = A$ and $\sum b_n = B$ are convergent series, then

(a) Sum Rule: ______________________________________

(b) Difference Rule: ______________________________________

(c) Constant Multiple Rule: _________________________, (any number k)

2.4 $\sum(a_n + b_n)$ can converge when $\sum a_n$ and $\sum b_n$ diverge.
Example: $\sum a_n =$ _____________________, $\sum b_n =$ ___________________________,
$\sum(a_n + b_n) =$ __________________

2.5 Adding or Deleting Terms: we can add or delete a _________ number of terms without altering the series'convergence or divergence, although in the case of convergence this will usually change the sum.

2.6 Reindexing:

(a) $\sum_{n=1}^{\infty} a_n =$ ______________________________

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then ____________ converges for any $k > 1$.

(c) If $\sum_{n=k}^{\infty} a_n$ converges for any $k > 1$ then ____________ converges.

(d) $\sum_{n=1}^{\infty} a_n =$ ______________ = ______________ = ____________________

(e) Example:

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \underline{\qquad\qquad} = \underline{\qquad\qquad} = \underline{\qquad\qquad}$$

Ex. 5 .... (example7, p548)

Test the divergences of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} n^2$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{-n}{2n+5}$

Ex. 6 .... (example9, p550)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}} =$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{2^n} =$

實習課練習 (EXERCISE 10.2)

☐ Find the sum of the series if the series converges.

7. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$.

13. $\sum_{n=0}^{\infty} (\frac{1}{2^n} + \frac{(-1)^n}{5^n})$.

44. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$.

45. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

47. $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)}) - \frac{1}{\ln(n+1)} \right)$

☐ Express each of the number as the ratio of two integers.

20. $0.\overline{234} = 0.234234234...$

25. $1.24\overline{123} = 1.24123123...$

☐ Use the nth-Term Test for divergence to show that the series is divergent, or state that the test is inconclusive.

30. $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$

33. $\sum_{n=1}^{\infty} \ln \frac{1}{n}$

☐ Which series converge and which diverge? If a series converges, find its sum.

49. $\sum_{n=0}^{\infty} (\frac{1}{\sqrt{2}})^n$

54. $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

58. $\sum_{n=0}^{\infty} \frac{1}{x^n}, \quad |x| > 1.$

60. $\sum_{n=1}^{\infty}(1-\frac{1}{n})^n$

67. $\sum_{n=0}^{\infty}(\frac{e}{\pi})^n$

☐ Find the values of x for which the given geometric series converges.

75. $\sum_{n=0}^{\infty}(-1)^n(x+1)^n$

78. $\sum_{n=0}^{\infty}(\ln x)^n$

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10.3 The Integral Test

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1 Nondecreasing Partial Sums

1.1 Given a series we have two questions: (a) Does the series ____________? (b) If it converges, what is its ________?

1.2 Suppose that $\sum_{n=1}^{\infty} a_n$ is an infinite series with $a_n \geq 0$ for all n. Then each partial sum is greater than or equal to its predecessor because __________________. The partial sums form a __________________________: _______________________________________

1.3 *Corollary of Theorem 6*

> A series $\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if its ________________ are _______________________.

1.4 The ___________________

$$\sum_{n=1}^{\infty} \frac{1}{n} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

is divergent, but this doesn't follow from the nth-Term Test.
The nth term $1/n$ does go to zero, but the series still diverges. The reason it diverges is because there is no upper bound for its partial sums.

1.5 *Theorem 9: The Integral Test*

> Let $\{a_n\}$ be a sequence of __________________. Suppose that ______________, where f is a _______________, ____________, _______________ function of x for all $x \geq N$ (N a positive integer). Then the series _____________ and the integral _______________ both converge or both diverge.

(說明如下:)

✎ Ex. **1** .... (`example2, p553`)

Does $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges?

sol:

✎ Ex. **2** .... (`example3, p555`)

Show that the **p-series**

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$$

converges if $p > 1$, and diverges if $p \leq 1$.

sol:

Ex. **3** .... (example4, p555)

Does $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converge?

sol:

實習課練習 (EXERCISE 10.3)

☐ Use the Integral Test to determine if the series converge or diverge.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$.

6. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

9. $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$.

☐ Determining convergence or divergence.

15. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$.

19. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$.

22. $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$.

27. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$.

28. $\sum_{n=1}^{\infty} (1+\frac{1}{n})^n$.

30. $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$.

36. $\sum_{n=1}^{\infty} \frac{2}{1+e^n}$.

38. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

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10.4 Comparison Tests

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1 Comparison Tests

1.1 *Theorem 10: The Comparison Test*

Let $\sum a_n$ be a series with ______________ terms.

(a) $\sum a_n$ ______________ if there is a ______________ series $\sum c_n$ with ____________ for all $n > N$, for some integer N.

(b) $\sum a_n$ ____________ if there is a _____________ series of nonnegative terms $\sum d_n$ with ____________ for all $n > N$, for some integer N.

✎ Ex. 1 .... (example1, p559)

(a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{5}{5n-1}$ converge?

(b) Does $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$ converge?

(c) Does $5 + \frac{2}{3} + \frac{1}{7} + 1 + \frac{1}{2+\sqrt{1}} + \cdots + \frac{1}{2^n + \sqrt{n}} + \cdots$ converge?

2 The Limit Comparison Test

2.1 *Theorem 11: Limit Comparison Test*

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

(a) If ____________________ then ________________ both converge or both diverge.

(b) If ________________ and __________ converges, then __________ converges.

(c) If _________________ and __________ diverges, then __________ diverges.

Ex. **2** .... (`example2, p560`)

(a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ converge?

(b) Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converge?

(c) Does $\displaystyle\sum_{n=2}^{\infty} \frac{1 + n\ln n}{n^2 + 5}$ converge?

✎ Ex. **3** .... (example3, p561)

Does $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge?

sol:

實習課練習 (EXERCISE 10.4)

☐ Use the Comparison Test to determine if each series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+30}$.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$.

☐ Use the Limit Comparison Test to determine if each series converges or diverges.

10. $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$.

14. $\displaystyle\sum_{n=1}^{\infty} (\frac{2n+3}{5n+4})^n$.

☐ Determining convergence or divergence?

17. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

THOMAS' CALCULUS (12/E)

10.5 The Ratio and Root Tests

開課班級: 通訊 1/電機 1/電資院 1 微積分
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1 The Ratio Test

1.1 *Theorem 12: The Ratio Test*

> Let $\sum a_n$ be a series with positive terms and suppose that ________
> Then
>
> (a) the series converges if ________,
>
> (b) the series diverges if ________ or ____________,
>
> (c) the test is inconclusive if ________.

Ex. 1 . . . . (`example1, p564`)

Investigate the convergence of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$

2 The Root Test

2.1 *Theorem 13: The Root Test*

Let $\sum a_n$ be a series with $a_n \geq 0$ and suppose that ________________ Then

(a) the series converges if __________,

(b) the series diverges if __________ or ________________,

(c) the test is inconclusive if __________.

Ex. 2 .... (example3, p566)

Investigate the convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} (\frac{1}{1+n})^n$

實習課練習 (EXERCISE 10.5)

☐ Use the Ratio Test to determine if each series converges or diverges.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$.

6. $\displaystyle\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$.

☐ Use the Root Test to determine if each series converges or diverges.

12. $\displaystyle\sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{8}{(3+(1/n))^{2n}}$.

☐ Determining convergence or divergence.

18. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.25^n}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

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10.6 Alternating Series, Absolute and Conditional Convergence

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1 Alternating Series

1.1 A series in which the terms are alternately __________ and __________ is an ________________.

(a) alternating harmonic series: ______________________________

(b) geometric series with $r = -1/2$: ______________________________

(c) $1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1} n + \cdots$

1.2 *Theorem 14: The Alternating Series Test (Leibniz's Theorem)*

The series

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if all three of the following conditions are satisfied:

(a) _______ are all __________.

(b) ____________ for all $n \geq N$, for some integer N.

(c) __________.

Ex. 1 .... (`example1, p569`)

Does the alternating harmonic series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converge?

sol:

2 Absolute and Conditional Convergence

2.1 *Definitions: Absolutely Convergent*

A series ____________ (is ____________) if the corresponding series of absolute values, ____________

2.2 *Definitions: Conditionally Convergent*

A series that converges but does not converge absolutely converges ____________.

2.3 The geometric series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$ converges ____________ because ____________ converges.

2.4 The alternating hramonic series does not converge absolutely. It converges ____________.

2.5 *Theorem 16: The Absolute Convergence Test*

If ____________ converges, then ____________ converges.

Ex. 2 .... (`example4, p571`)

(a) Does $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ converge absolutely?

(b) Does $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converge absolutely?

✎ Ex. **3** .... (example5, p571)

Determine the convergence (absolutely convergence, conditionally convergence) of the alternating p-series: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$.

sol:

3 Summary of Tests for Convergence and Divergence of Series

3.1 The nth-Term Test: ______________________.

3.2 Geometric series: ______________________.

3.3 p-series: ______________________.

3.4 Series with nonnegative terms:
Try ______________________.
Try ______________________.

3.5 Series with some negative terms: ______________________.

3.6 Alternating series: ______________________.

實習課練習 (EXERCISE 10.6)

☐ Determining convergence or divergence.

2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$

11. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

13. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

☐ Which of the series converge absolutely, which converge and which diverge?

16. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

20. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

25. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$

30. $\displaystyle\sum_{n=2}^{\infty}(-1)^{n}\frac{\ln n}{n-\ln n}$

39. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{(2n)!}{2^n n! n}$

42. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}(\sqrt{n^2+n}-n)$

44. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\sqrt{n}+\sqrt{n+1}}$

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10.7 Power Series

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1 Power Series, The Convergence Theorem

1.1 *Definitions: Power Series, Center, Coefficients*

A power series about __________ is a series of the form

____

A power series about __________ is a series of the form

____

in which the center a and the coefficients $c_0, c_1, c_2, \cdots, c_n$ are constants.

2 The Radius of Convergence of a Power Series

2.1 *Theorem 18: The Convergence Theorem for Power Series*

If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = c \neq 0$,then it converges absolutely for all x with ____________. If the series diverges for $x = d$, then it diverges for all x with ____________.

2.2 *COROLLARY TO THEOREM 18*

The convergence of the series $\sum c_n(x-a)^n$ is the one of the following three:

(a) There is a positive number R such that the series diverges for x with ____________ but converges absolutely for x with ____________. The series may or may not converge at either of the endpoints ___________ and ___________.

(b) The series converges absolutely for ________________.

(c) The series converges at ________ and diverges _________________.

2.3 R is called the ___________________ of the power series and the interval of radius R centered at $x=a$ is called the interval of convergence.

2.4 *How to Test a Power Series for Convergence*

(a) Use the ________ Test (or ___________ Test) to find the interval where the series converges ___________. Ordinarily, this is an open interval ____________ *or* ___________________

(b) If the interval of absolute convergence is ________, test for convergence or divergence at ______________. Use a ____________ Test, the __________ Test, or the _________________ Test.

(c) If the interval of absolute convergence is __________________, the series diverges for ____________ (it does not even converge conditionally), because the nth term does not approach zero for those values of x.

Ex. 1 .... (example1, p575)

Find the sum of a geometric power series with $a=1$, $r=x$:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots$$

sol:

✎ Ex. **2** .... (`example2, p576`)

Find the sum of a geometric power series with $a = 1$, $r = -(x-2)/2$:
$1 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2 + \cdots + (-\frac{1}{2})^n(x-2)^n + \cdots$

sol:

✎ Ex. **3** .... (`example3(a), p576`)

For what values of x do the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converge?

sol:

✎ Ex. **4** .... (`example3(b), p577`)

For what values of x do the power series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$ converge?

sol:

✐ Ex. **5** .... (`example3(c), p577`)

For what values of x do the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge?

sol:

✐ Ex. **6** .... (`example3(d), p577`)

For what values of x do the power series $\displaystyle\sum_{n=0}^{\infty} n!x^n$ converge?

sol:

實習課練習 (EXERCISE 10.7)

☐ Find the series's radius and interval of convergence. Foe what values of x does the series converge absolutely, or conditionally?

3. $\sum_{n=0}^{\infty}(-1)^n(4x+1)^n$

8. $\sum_{n=0}^{\infty}\frac{(-1)^n(x+2)^n}{n}$

11. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$

15. $\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n^2+3}}$

23. $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$

34. $\sum_{n=1}^{\infty}\frac{3\cdot 5\cdot 7\cdots(2n+1)}{n^2\cdot 2^n}x^{n+1}$

36. $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$

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10.8 Taylor and Maclaurin Series

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1 Series Representations

1.1 Assume that $f(x)$ is the sum of a power series

$$
\begin{aligned}
f(x) &= \underline{\hspace{6cm}} \\
&= \underline{\hspace{14cm}}
\end{aligned}
$$

$$
\begin{aligned}
f'(x) &= \underline{\hspace{15cm}} \\
f''(x) &= \underline{\hspace{12cm}} \\
f'''(x) &= \underline{\hspace{14cm}} \\
&\vdots \\
f^{(n)}(x) &= \underline{\hspace{13cm}}
\end{aligned}
$$

1.2 $f^{(n)}(a) = \underline{\hspace{2.5cm}} \Rightarrow \underline{\hspace{4.5cm}}$

1.3 $f(x) = \underline{\hspace{17cm}}$

1.4 *Definitions: Taylor Series, Maclaurin Series*

> Let f be a function with derivatives of all orders throughout some interval containing a as an interior point.
>
> (a) Then the ________________ generated by f at $x = a$ is ______________
>
> ____

(a) The ____________________ generated by f is

____

____

the Taylor series generated by f at $x = 0$.

Ex. **1** .... (`example1, p585`)

Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where, if anywhere, does the series converge to $1/x$?

sol:

2 Taylor Polynomials

2.1 The linearization of a differentiable function f at a point a is the polynomial of degree one given by

$$P_1(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

We used this ________________ to approximate $f(x)$ at values of x near a.

2.2 *Definitions: Taylor Polynomial of Order n*

Let f be a function with derivatives of order k for $k = 1, 2, \cdots, N$ in some interval containing a as an interior point. Then for any integer n from 0 through N, the ________________________________ generated by f at $x = a$ is the polynomial

$P_n(x) =$ ____

✎ Ex. **2** .... (example2, p586)

Find the Taylor series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.

sol:

✎ Ex. **3** .... (example3, p587)

Find the Taylor series and the Taylor polynomials generated by $f(x) = \cos x$ at $x = 0$.

sol:

實習課練習 (EXERCISE 10.8)

☐ Find the Taylor polynomials of order 0,1,2, and 3 generated by f at a.

5. $f(x) = \dfrac{1}{x},\ a = 2$

10. $f(x) = \sqrt{1-x},\ a = 0$

☐ Find the Maclaurin series.

12. xe^x

15. $\sin 3x$

☐ Find the Taylor series generated by f at $x = a$.

23. $f(x) = x^3 - 2x + 4,\ a = 2$

27. $f(x) = 1/x^2,\ a = 1$

30. $f(x) = 2^x,\ a = 1$

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11.3 Polar Coordinates

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1 Definition of Polar Coordinates

1.1 *Definitions: Polar Coordinates*

> Fix an ______ (called the ______) and an initial ______ from O. Then each point P can be located by assigning to it a ______ pair ______ in which r gives the ______ from O to P and θ gives the ______ from the initial ray to ray $\bar{OP}$.

1.2 圖示如下:

(a) Polar coordinates (b) (r, θ) are not unique. (c) (r, θ) can have negative r-values.

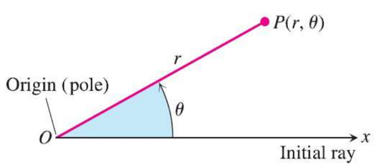

FIGURE 11.18 To define polar coordinates for the plane, we start with an origin, called the pole, and an initial ray.

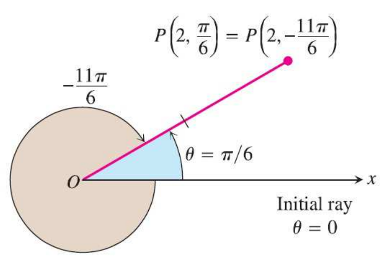

FIGURE 11.19 Polar coordinates are not unique.

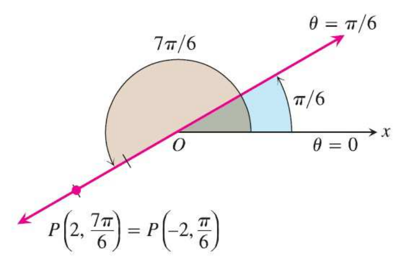

FIGURE 11.20 Polar coordinates can have negative r-values.

1.3 One pair of cartesian coordinates imples ______ pairs of polar coordinates.

1.4 Polar Equations and Graphs (I)

(a) If we hold r fixed at a constant value ______, the point ______ will lie ______ units from the origin O.

(b) As θ varies over any interval of length ______, P then traces a circle of ______ centered at O.

(c) If we hold θ fixed at a constant value __________ and let r vary between _______________, the point $P(r, \theta)$ traces the line through O that makes an angle of measure θ_0 with the initial ray.

1.5 Polar Equations and Graphs (II)

(a) Equation: $r = a$, Graph: ________________________________

(b) Equation: $\theta = \theta_0$, Graph: ____

(c) $r = 1$ and $r = -1$ are equations for the ________________________________.

✎ Ex. **1** .... (`example1, p627`)

Find all the polar coordinates of the point $P(2, \pi/6)$

sol:

✎ Ex. **2** .... (`example3, p628`)

Graph the sets of points whose polar coordinates satisfy the following conditions.

(a) $1 \le r \le 2$ and $0 \le \theta \le \dfrac{\pi}{2}$

(b) $-3 \le r \le 2$ and $\theta = \dfrac{\pi}{4}$

(d) $\dfrac{2\pi}{3} \le \theta \le \dfrac{5\pi}{6}$

2 Relating Polar and Cartesian Coordinates

2.1 *Equations Relating Polar and Cartesian Coordinates*

$$x = r\cos\theta, \quad y = r\sin\theta, \quad x^2 + y^2 = r^2$$

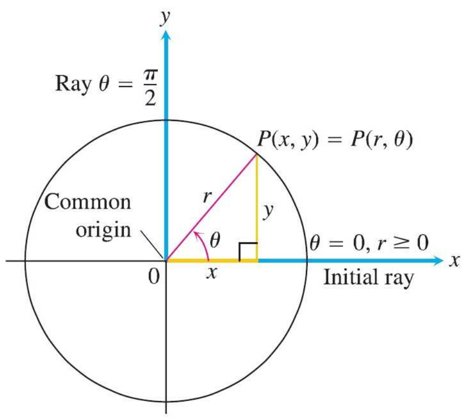

FIGURE 11.24 The usual way to relate polar and Cartesian coordinates.

Ex. **3** .... (example5, p629)

Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

sol:

Ex. **4** .... (example6, p629)

Replace the following polar equations by equivalent Cartesian equations, and identify their graphs.

(a) $r\cos\theta = -4$

(b) $r^2 = 4r\cos\theta$

(c) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

sol:

實習課練習 (EXERCISE 11.3)

☐ Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point.

3. $(2, \pi/2)$, $(2, 0)$, $(-2, \pi/2)$, $(-2, 0)$.

4. $(3, \pi/4)$, $(-3, \pi/4)$, $(3, -\pi/4)$, $(-3, -\pi/4)$.

☐ Replace the following polar equation by equivalent Cartesian equations.

30. $r \cos\theta = 0$

33. $r \cos\theta + r \sin\theta = 1$

38. $r^2 \sin 2\theta = 2$

44. $\cos^2\theta = \sin^2\theta$

49. $r = 2\cos\theta + 2\sin\theta$

☐ Replace the following Cartesian equation by equivalent polar equations.

54. $y = 1$

57. $x^2 + y^2 = 4$

60. $xy = 2$

62. $x^2 + xy + y^2 = 1$

65. $(x-3)^2 + (y+1)^2 = 4$

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11.4 Graphing in Polar Coordinates

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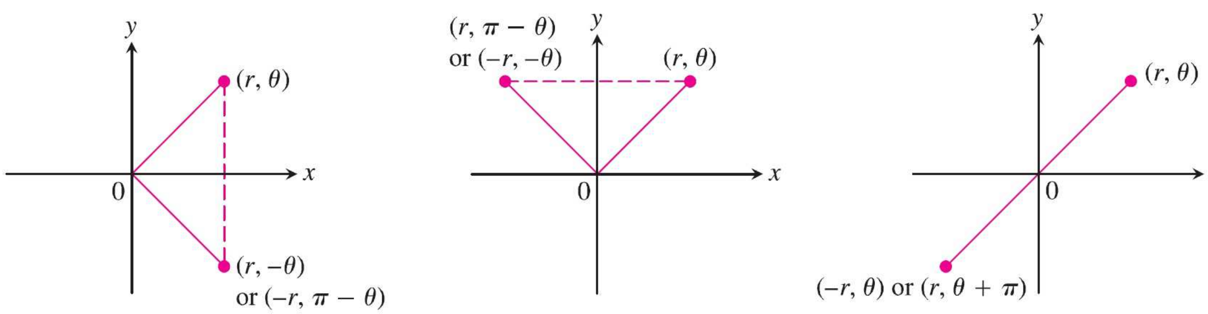
1 Symmetry and Slope

1.1 *Symmetry Tests for Polar Graphs*

(a) 1. Symmetry about the __________:
If the point (r, θ) lies on the graph, the point __________ or __________ lies on the graph.

(b) 2. Symmetry about the __________:
If the point (r, θ) lies on the graph, the point __________ or __________ lies on the graph.

(c) 3. Symmetry about the __________:
If the point (r, θ) lies on the graph, the point __________ or __________ lies on the graph.



(a) About the x-axis (b) About the y-axis (c) About the origin

1.2 *Slope*

The slope of a polar curve $r = f(\theta)$ is given by __________, not by __________.

1.3 *Slope of the Curve* $r = f(\theta)$

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \underline{\hspace{6cm}}, \text{ provided } dy/d\theta \neq 0 \text{ at } (r,\theta).$$

1.4 Consider the graph of f as the graph of the parametric equations $x = r\cos\theta = f(\theta)\cos\theta, y = r\sin\theta = f(\theta)\sin\theta$.

$\frac{dy}{dx} =$

1.5 If the curve $r = f(\theta)$ passes through the origin at $\theta = \theta_0$, then $f(\theta_0) = 0$ and the slope equation gives

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \underline{\hspace{6cm}}$$

1.6 If the graph of $r = f(\theta)$ passes through the origin at $\theta = \theta_0$ the value the slope of the curve there is __________.

1.7 The reason we say ____________________ and not just _______________________ is that a polar curve may pass through the origin (or any point) ___________________, with different slopes at different.

✎ **Ex. 1** .... (`example1, p632`)

Graph the curve $r = 1 - \cos\theta$.

sol:

Ex. 2 .... (example2, p633)

Graph the curve $r^2 = 4\cos\theta$.

sol:

Ex. 3 .... (example3, p3)722

Graph the curve $r^2 = \sin 2\theta$.

sol:

2 重要極座標圖形

2.1 圓 circle

$r = a\cos\theta$		$r = a\sin\theta$	
$a < 0$	$a > 0$	$a < 0$	$a > 0$

2.2 心臟線 Cardioid

$r = a(1 \pm \cos\theta)$		$r = a(1 \pm \sin\theta)$	
$a < 0$	$a > 0$	$a < 0$	$a > 0$

2.3 蚶線 LimaCon

$r = a \pm b\cos\theta$		$r = a \pm b\sin\theta$	
$a > b$	$a < b$	$a > b$	$a < b$

2.4 雙紐線 Lemniscates

$r^2 = a^2 \cos 2\theta$	$r^2 = a^2 \sin 2\theta$

2.5 玫瑰線 Rose curve

$r = a \cos n\theta$		$r = a \sin n\theta$	
$n : odd$	$n : even$	$n : odd$	$n : even$

實習課練習 (EXERCISE 11.4)

2. Graph the curve $r = 2 - 2\cos\theta$.

6. Graph the curve $r = 1 + 2\sin\theta$.

13. Graph the curve $r^2 = 4\cos 2\theta$.

19. Graph the curve $r = \sin 2\theta$.

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11.5 Areas and Lengths in Polar Coordinates

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1 Area in the Plane

1.1 The fan-shaped circular sector has radius ____________ and central angle of radian measure _______. The area is _________ times the area of a circle of radius ______, or

$$A_k = \underline{\hspace{6cm}}, \quad \sum_{k=1}^{n} A_k = \underline{\hspace{5cm}}$$

$$A = \underline{\hspace{6cm}} = \underline{\hspace{4cm}}$$

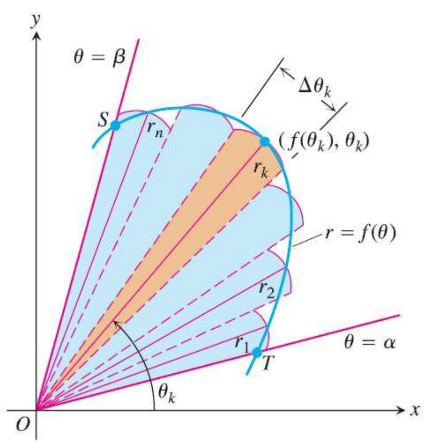

FIGURE 11.30 To derive a formula for the area of region *OTS*, we approximate the region with fan-shaped circular sectors.

1.2 *Area of the Fan-Shaped Region Between the Origin and the Curve* $r = f(\theta)$, $\alpha < \theta < \beta$

$$A = \underline{\hspace{4cm}}.$$

This is the integral of the area differential $dA = \underline{\hspace{3cm}} = \underline{\hspace{4cm}}$.

1.3 *Area of the Region* $0 \le r_1(\theta) \le r \le r_2(\theta)$, $\alpha \le \theta \le \beta$

$$A = \underline{\hspace{7cm}} = \underline{\hspace{5cm}}$$

✎ Ex. 1 .... (`example1, p636`)

Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

sol:

Ex. 2 .... (example2, p636)

Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.

sol:

Ex. 3 .... (example3, p637)

Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$.

sol:

實習課練習 (EXERCISE 11.5)

3. Find the area of the region inside the oval limacon $r = 4 + 2\cos\theta$.

8. Find the area of the region inside the six-leaved rose $r^2 = 2\sin 3\theta$.

13. Find the area of the region inside the lemniscate $r^2 = 6\cos 2\theta$ and outside the circle $r = \sqrt{3}$.

14. Find the area of the region inside the circle $r = 3a\cos\theta$ and outside the cardioid $r = a(1 + \cos\theta),\ a > 0$.

THOMAS' CALCULUS (12/E)

12.2 Vectors

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1 Component Form

1.1 *Definition*

The vector represented by the ____________________ has initial point ______ and terminal point ______ and its length is denoted by ________. Two vectors are equal if they have the ____________ and __________.

Definition

(a) If $\vec{v}$ is a two-dimensional vector in the plane equal to the vector with initial point at the _________ and terminal point __________, then the component form of ______ is ______________.

(b) If $\vec{v}$ is a three-dimensional vector in the plane equal to the vector with initial point at the _________ and terminal point ____________, then the component form of $\vec{v}$ is ________________.

1.2 The magnitude or length of the vector $\vec{v} = \vec{PQ}$, $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$, is the nonnegative number

$$\|\vec{v}\| = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

✎ Ex. 1 .... (example1, p666)

Find the component form and length of the vector with initial point $P(-3, 4, 1)$ and terminal $Q(-5, 2, 2)$.

sol:

2 Vector Algebra Operations

2.1 *Definition*

Let $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ be vectors with k a scalar.

(a) Addition: $\vec{u} + \vec{v} =$ ______________________________

(b) Scalar multiplication: $k\vec{u} =$ ____________________

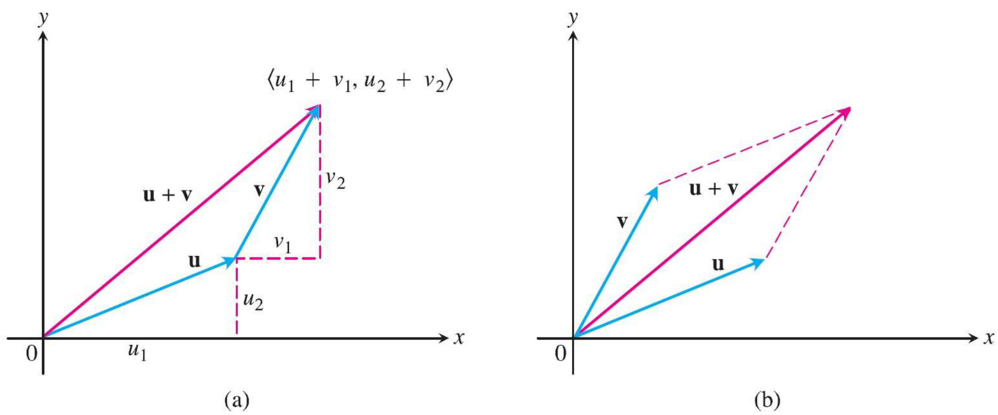

FIGURE 12.12 (a) Geometric interpretation of the vector sum. (b) The parallelogram law of vector addition.

2.2 *Properties of Vector Operations*

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors and a, b be scalars.

1. $\vec{u} + \vec{v} =$ __________ 2. $(\vec{u} + \vec{v}) + \vec{w} =$ _______________

3. $\vec{u} + \vec{0} =$ ______ 4. $\vec{u} + (-\vec{u}) =$ ______

5. $0\vec{u} = \vec{0}$ 6. $1\vec{u} = \vec{u}$

7. $a(b\vec{u}) =$ __________ 8. $a(\vec{u} + \vec{v}) =$ ____________

9. $(a + b)\vec{u} =$ ____________

2.3 A vector $\vec{v}$ of length 1 is called ______________.

2.4 The standard unit vector are _________________, _________________, and _________________.

2.5 Any vector $\vec{v} =< v_1, v_2, v_3 >$ can be written as a linear combination of the standard unit:

$$\vec{v} = \underline{\hspace{8cm}}$$
$$= \underline{\hspace{7cm}}$$
$$= \underline{\hspace{3cm}}$$

2.6 The scalar ______ is the ______ (j-component, k-component) of the vector $\vec{v}$.

2.7 The vector from $P(x_1, y_1, z_1)$ to $Q(x2, y_2, z_2)$ is

$$\vec{PQ} = \underline{\hspace{6cm}}$$

2.8 Whenever $\vec{u} \neq \vec{0}$, ______ is a unit vector in the direction of $\vec{v}$.

2.9 The equation $\vec{v} =$ ______ expresses $\vec{v}$ as its length times its direction.

2.10 The midpoint M of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point ______.

Ex. 2 .... (example3, p668)

Let $\vec{u} =< -1, 3, 1 >$ and $\vec{v} =< 4, 7, 0 >$. Find the component of (a) $2\vec{u} + 3\vec{v}$ (b) $\vec{u} - \vec{v}$ (c) $\|\frac{1}{2}\vec{u}\|$.

sol:

Ex. 3 .... (example4, p669)

Find a unit vector $\vec{u}$ in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

sol:

實習課練習 (EXERCISE 12.2)

☐ In Exercise 17-22, express each vector in the form $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$.

18. $P_1\vec{P}_2$ if $\vec{P_1}$ is the point $(1, 2, 0)$ and P_2 is the point $(-3, 0, 5)$.

22. $-2\vec{u} + 3\vec{v}$ if $\vec{u} =< -1, 0, 2 >$ and $\vec{v} =< 1, 1, 1 >$.

25. Express $2\vec{i} + \vec{j} - 2\vec{k}$ as a product of its length and direction.

33. Find a vector of magnitude 7 in the direction of $\vec{v} = 12\vec{i} - 5\vec{k}$.

THOMAS' CALCULUS (12/E)

12.3 The Dot Product

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1 Angle Between Vectors

1.1 *Definition*

> The dot product $\vec{u}\cdot\vec{v}$ of vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is
>
> $$\vec{u}\cdot\vec{v} = \underline{\hspace{6cm}}$$

1.2 *Theorem 1*

> The angle θ between two nonzero vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is given by
>
> $$\theta = \underline{\hspace{8cm}} = \underline{\hspace{5cm}}$$

Ex. 1 .... (`example2, p676`)

Find the angle between $\vec{u} = \vec{i} - 2\vec{j} - 2\vec{k}$ and $\vec{v} = 6\vec{i} + 3\vec{j} + 2\vec{k}$.

sol:

✎ Ex. 2 .... (example3, p676)

Find the angle θ in the triangle ABC determined by the vertices $A = (0,0)$, $B = (3,5)$ and $C = (5,2)$.

sol:

2 Perpendicular (Orthogonal) Vectors

2.1 *Definition*

Vectors $\vec{u}$ and $\vec{v}$ are ____________ (or ______________) if and only if __________.

2.2 *Properties of the Dot Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and c is a scalar, then

1. $\vec{u} \cdot \vec{v} =$ _______
2. $(c\vec{u}) \cdot \vec{v} =$ _________ $=$ _________
3. $\vec{u} \cdot (\vec{v} + \vec{w}) =$ _____________
4. $\vec{u} \cdot \vec{u} =$ _______
5. $\vec{0} \cdot \vec{u} =$ ____

2.3 The vector projection of $\vec{u} = \vec{PQ}$ onto a nonzero vector $\vec{v} = \vec{PS}$ is the vector $\vec{PR}$ determined by dropping a perpendicular from Q to the line PS:

____

2.4 The scalar component of $\vec{u}$ in the direction $\vec{v}$ is the scalar

___________ $=$ ________ $=$ __________

Ex. **3** .... (example5, p678)

Find the vector projection of $\vec{u} = 6\vec{i} + 3\vec{j} + 2\vec{k}$ onto $\vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$ and the scalar component of $\vec{u}$ in the direction of $\vec{v}$.

sol:

實習課練習 (EXERCISE 12.3)

☐ Find

(a) $\vec{v} \cdot \vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$

(b) the cosine of the angle between $\vec{v}$ and $\vec{u}$.

(c) the scalar component of $\vec{u}$ in the direction of $\vec{v}$

(d) the vector $\text{proj}_{\vec{v}}\vec{u}$

4. $\vec{v} = 2\vec{i} + 10\vec{j} - 11\vec{k}$, $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$

8. $\vec{v} =< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} >$, $\vec{u} =< \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} >$

THOMAS' CALCULUS (12/E)

12.4 The Cross Product

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1 The Cross Product of Two Vectors in Space

1.1 *Definition*

Let the unit vector $\vec{n}$ be perpendicular to the plane by the right-hand rule. Then the ________________ __________ ($\vec{u}$ cross $\vec{v}$) is the vector defined by

$$\vec{u} \times \vec{v} = \underline{\hspace{4cm}}$$

1.2 Nonzero vectors $\vec{u}$ and $\vec{v}$ are ___________ if and only if _____________.

1.3 *Properties of the Cross Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and r, s are scalars, then

1. $(r\vec{u}) \times (s\vec{v}) =$ ______________
2. $\vec{u} \times (\vec{v} + \vec{w}) =$ ________________
3. $\vec{v} \times \vec{u} =$ ____________
4. $(\vec{v} + \vec{w}) \times \vec{u} =$ ________________
5. $\vec{0} \times \vec{u} =$ _____
6. $\vec{u} \times (\vec{v} \times \vec{w}) =$ ____________________

1.4 (a) $\vec{i} \times \vec{j} =$ _______________

(b) $\vec{j} \times \vec{k} =$ _______________

(c) $\vec{k} \times \vec{i} =$ _______________

(d) $\vec{i} \times \vec{i} =$ ____________________

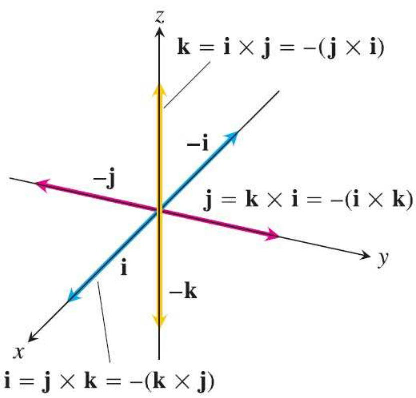

FIGURE 12.29 The pairwise cross products of **i**, **j**, and **k**.

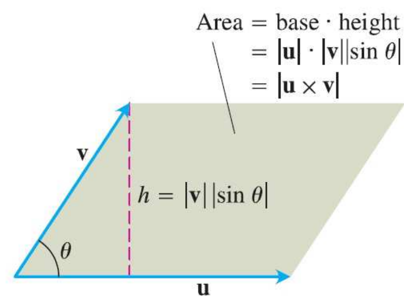

FIGURE 12.30 The parallelogram determined by **u** and **v**.

1.5 If $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$ and $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$, then

$$\vec{u} \times \vec{v} =$$

____

pf.

1.6 Calculating the triple scalar product as a determinnat.

$$(\vec{u} \times \vec{v}) \cdot \vec{w} =$$

____

pf.

1.7 The magnitude of $\vec{u} \times \vec{v}$ is the area of the ________________ determined by $\vec{u}$ and $\vec{v}$, $\vec{u}$ being the ________ of the parallelogram and $\|\vec{v}\|\|\sin\theta\|$ the __________.

$$\|\vec{u} \times \vec{v}\| = \underline{\hspace{5cm}} = \underline{\hspace{4cm}}$$

✎ Ex. **1** . . . . (example1, p684)

Find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ if $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = -4\vec{i} + 3\vec{j} + \vec{k}$.

sol:

✎ Ex. **2** . . . . (example2, p684)

Find a vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

✎ Ex. **3** . . . . (example4, p684)

Find a unit vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

實習課練習 (EXERCISE 12.4)

☐ In Exercise 1-8, find the length and direction of $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

3. $\vec{u} = 2\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{v} = -\vec{i} + \vec{j} - 2\vec{k}$

6. $\vec{u} = \vec{i} \times \vec{j}$, $\vec{v} = \vec{j} \times \vec{k}$

☐ In Exercise 15-18, (a) Find the area of the triangle determined by the points P, Q and R. (b) Find a unit vector perpendicular to plane PQR.

16. $P(1, 1, 1)$, $Q(2, 1, 3)$, $R(3, -1, 1)$.

THOMAS' CALCULUS (12/E)

14.2 Limits and Continuity in Higher Dimensions

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1 Limits for Functions of Two Variables

1.1 *Definition*

We say that a function __________ approaches the limit ______ as __________ approaches __________, and write

____

if, for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all (x, y) in the domain of f,

__________________ whenever ______________________________.

1.2 *Theorem 1: Properties of Limits of Functions of Two Variables*

The following rules hold if L, M and k are real numbers and

$$\lim_{(x,y)\to(x_0),y_0)} f(x,y) = \_\_\_\_\_\_ \quad \text{and} \quad \lim_{(x,y)\to(x_0),y_0)} g(x,y) = \_\_\_\_\_\_$$

(a) *Sum Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) + g(x,y)) =$ __________

(b) *Difference Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) - g(x,y)) =$ __________

(c) *Constant Multiple Rule:* $\lim_{(x,y)\to(x_0),y_0)} kf(x,y) =$ ______

(d) *Product Rule:* $\lim_{(x,y)\to(x_0),y_0)} (f(x,y) \cdot g(x,y)) =$ _________

(e) *Quotient Rule:* $\lim_{(x,y)\to(x_0),y_0)} \frac{f(x,y)}{g(x,y))} = \_\_\_\_\_\_,\quad M \neq 0.$

(f) *Power Rule:* $\lim_{(x,y)\to(x_0),y_0)} [f(x,y)]^n = \_\_\_\_\_\_, n \in N^+.$

(g) *Root Rule:* $\lim_{(x,y)\to(x_0),y_0)} \sqrt[n]{f(x,y)} = \_\_\_\_\_\_, n \in N^+.$

Ex. 1 .... (`example1, p757`)

(a) $\lim_{(x,y)\to(0,1)} \frac{x - xy + 3}{x^2y + 5xy - y^3} =$

(b) $\lim_{(x,y)\to(3,-4)} \sqrt{x^2 + y^2} =$

Ex. 2 .... (`example2, p757`)

Find $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

sol:

2 Continuity

2.1 *Definition*

A function $f(x,y)$ is ________ at the point (x_0, y_0) if

(a) f is ________ at (x_0, y_0),

(b) ____________ exists,

(c) ________________.

A function is ________ if it is continuous at every point of its domain.

2.2 *Two-Path Test for Nonexistence of a Limit*

If a function $f(x, y)$ has a different limits along ____________________ in the domain of f as (x, y) approaches (x_0, y_0), then $\lim_{(x,y)\to(x_0,y_0)} f(x, y)$ does not exist.

實習課練習 (EXERCISE 14.2)

2. $\lim\limits_{(x,y)\to(0,4)} \dfrac{x}{\sqrt{y}}$

3. $\lim\limits_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$

7. $\lim\limits_{(x,y)\to(0,\ln 2)} e^{x-y}$

12. $\lim\limits_{(x,y)\to(\pi/2,0)} \dfrac{\cos y+1}{y-\sin x}$

14. $\lim\limits_{\substack{(x,y)\to(1,1)\\ x\neq y}} \dfrac{x^2-y^2}{x-y}$

18. $\lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \dfrac{x+y-4}{\sqrt{x+y}-2}$

21. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

22. $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-\cos xy}{xy}$

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14.3 Partial Derivatives

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1 Partial Derivatives of a Function of Two Variables

1.1 *Definition*

The ____________ of $f(x, y)$ with respect to x at the point (x_0, y_0) is

$$\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

provided the limit exists.

1.2 An equivalent expression

$$\_\_\_\_\_\_\_\_$$

1.3 Several notations

$$\_\_\_\_\_\_,\quad \_\_\_\_\_\_,\quad \_\_\_\_\_\_,\quad \_\_\_\_,\quad \_\_\_\_$$

1.4 The partial derivative of $f(x, y)$ with respect to ________ is obtained by differentiating f in the usual way while treating ________ as a ________.

Ex. 1 .... (example1, p766)

Find the values of $\partial f/\partial x$ and $\partial f/\partial y$ at the point $(4, -5)$ if $f(x, y) = x^2+3xy+y-1$.

sol:

✎ Ex. 2 .... (example2, p767)

Find $\partial f/\partial y$ as a function if $f(x, y) = y \sin xy$.

sol:

✎ Ex. 3 .... (example3, p767)

Find f_x and f_y as functions if $f(x, y) = \dfrac{2y}{y + \cos x}$.

sol:

✎ Ex. 4 .... (example4, p767)

Find $\partial z/\partial x$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivative exists.

sol:

✎ Ex. **5** .... (example6, p768)

Find $\partial f/\partial z$ if x, y and z are independent variables and $f(x,y,z) = x\sin(y+3z)$.

sol:

2 Second-Order Partial Derivatives

2.1 Differentiate $f(x,y)$ twice, we produce its second-order derivatives.

__________, __________, ____________________, __________

2.2 *Theorem 2: The Mixed Derivative Theorem*

> If $f(x,y)$ and its partial derivatives _______________ are defined throughout an open region containing a point (a,b) and are all continuous at (a,b), then _______________.

✎ Ex. **6** .... (example9, p770)

If $f(x,y) = x\cos y + ye^x$, find the second-order derivatives.

sol:

✎ Ex. **7** .... (example10, p770)

Find $\partial^2 w/\partial x \partial y$ if $w = xy + \dfrac{e^y}{y^2+1}$.

sol:

✎ Ex. **8** .... (example11, p771)

Find f_{yxyz} if $f(x, y, z) = 1 - 2xy^2z + x^2y$.

sol:

實習課練習 (EXERCISE 14.3)

7. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y)=\sqrt{x^2+y^2}$.

16. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y)=e^{xy}\ln y$.

21. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y)=\displaystyle\int_x^y g(t)\,dt$, ($g$ is continuous for all t).

22. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y)=\displaystyle\sum_{n=1}^{\infty}(xy)^n$.

25. Find f_x, f_y and f_z: $f(x,y,z)=x-\sqrt{y^2+z^2}$.

32. Find f_x, f_y and f_z: $f(x,y,z)=e^{-xyz}$.

43. Find all the second-order partial derivatives: $g(x,y)=x^y+\cos y+y\sin x$.

52. Verify that $w_{xy}=w_{yx}$: $w=e^x+x\ln y+y\ln x$.

66. Find the value of $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation $xz+y\ln x-x^2+4=0$ defines x as a function of the two independent variables y and z and the partial dervitive exists.

72. Let $f(x,y)=\begin{cases}\sqrt{x}, & x\geq 0\\ x^2, & x<0.\end{cases}$

Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

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14.4 The Chain Rule

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1 The Chain Rule

1.1 *Theorem 5: Chain Rule for Functions of Two Independent Variables*

If ______________ is differentiable and if ____________, ____________ are differentiable functions of t, then the composite ____________________ is a differentiable function of t and

$$\frac{dw}{dt} = \rule{6cm}{0.4pt}$$

1.2 *Theorem 6: Chain Rule for Functions of Three Independent Variables*

If $w = f(x, y, z)$ is differentiable and x, y, and z are differentiable functions of t, then the composite $w = f(x(t), y(t), z(t))$ is a differentiable function of t and

$$\frac{dw}{dt} = \rule{8cm}{0.4pt}$$

1.3 The Branch Diagram:

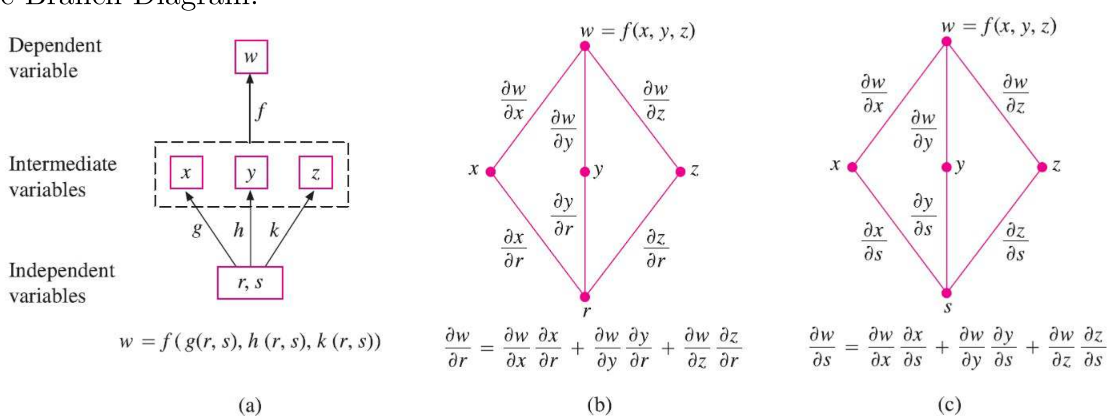

FIGURE 14.21 Composite function and branch diagrams for Theorem 7.

1.4 *Theorem 7: Chain Rule for Two Independent Variables and Three Intermediate Variables*

Suppose that $w = f(x, y, z)$, ____________, ____________ and ____________. If all four functions are differentiable, then w has partial derivatives with respect to r and s, given by

$$\frac{dw}{dr} = \underline{\hspace{6cm}}$$

$$\frac{dw}{ds} = \underline{\hspace{6cm}}$$

✎ Ex. **1** .... (example1, p776)

Use the Chain Rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t$, $y = \sin t$. What is the derivative's value at $t = \pi/2$.

sol:

✎ Ex. **2** .... (example2, p777)

Find dw/dt if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

sol:

Ex. 3 .... (example3, p778)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x+2y+z^2$, $x = \dfrac{r}{s}$, $y = r^2+\ln s$, $z = 2r$.

sol:

Ex. 4 .... (example3, p779)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

sol:

2 Implicit Differentiation

2.1 *Theorem 8: A Formula for Implicit Differentiation*

Suppose that $F(x, y)$ is differentiable and that the equation $F(x, y) = 0$ defines y as a differentiable function of x. Then at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = \underline{\hspace{3cm}}$$

pf.

2.2 Suppose that $F(x, y, z) = 0$ defines z implicitly as a function $z = f(x, y)$:

$$\frac{\partial z}{\partial x} = \underline{\hspace{2cm}}, \quad \frac{\partial z}{\partial y} = \underline{\hspace{2cm}}$$

Ex. 5 .... (example5, p780)

Use implicit differentiation to find dy/dx if $y^2 - x^2 - \sin xy = 0$.

sol:

Ex. 6 .... (example6, p781)

Find $\partial z/\partial x$ and $\partial z/\partial y$ at (0,0,0) if $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

sol:

實習課練習 (EXERCISE 14.4)

3. Evaluate dw/dt at the given value of t: $w = \dfrac{x}{z} + \dfrac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = 1/t$, $t = 3$.

5. Evaluate dw/dt at the given value of t: $w = 2ye^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \tan^{-1} t$, $z = e^t$, $t = 1$.

8. Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u, v): $z = \tan^{-1}(x/y)$, $x = u \cos v$, $y = u \sin v$, $(u, v) = (1.3, \pi/6)$.

9. Evaluate $\partial w/\partial u$ and $\partial w/\partial v$ at the given point (u, v): $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$, $(u, v) = (1/2, 1)$.

11. Evaluate $\partial u/\partial x$, $\partial u/\partial y$ and $\partial u/\partial z$ at the given point (x, y, z): $u = \dfrac{p - q}{q - r}$, $p = xy + yz + xz$, $q = x - y + z$, $r = x + y - z$, $(x, y, z) = (\sqrt{3}, 2, 1)$.

21. Draw a branch diagram and write a Chain Rule formula for each derivative. $\partial w/\partial s$ and $\partial w/\partial t$ for $w = g(u)$, $u = h(s, t)$.

28. Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$

29. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the given point: $z^3 - xy + yz + y^3 - 2 = 0$, $(1, 1, 1)$

35. Find $\partial w/\partial v$ when $u = 0$, $v = 0$ if $w = x^2 + (y/x)$, $x = u - 2v + 1$, $y = 2u + v - 2$.

THOMAS' CALCULUS (12/E)

14.5 Directional Derivatives and Gradient Vectors

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1 Directional Derivatives in the Plane

1.1 Suppose that the function __________ is defined throughout a region R in the xy-plane, that ____________ is a point in R, and that ________________ is a unit vector. Then the equations

$$x = \underline{\hspace{3cm}}, \quad y = \underline{\hspace{3cm}}$$

parametrize the line through P_0 parallel to $\vec{u}$.

1.2 If the parameter s measures ____________ from P_0 in the direction of _____, we find the rate of change of f at P_0 in the direction of $\vec{u}$ by calculating _________ at _______.

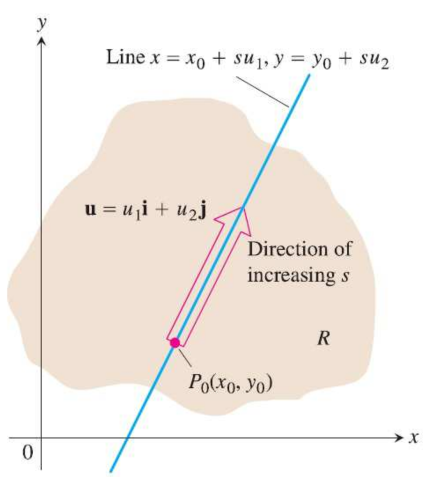

FIGURE 14.26 The rate of change of f in the direction of $\mathbf{u}$ at a point P_0 is the rate at which f changes along this line at P_0.

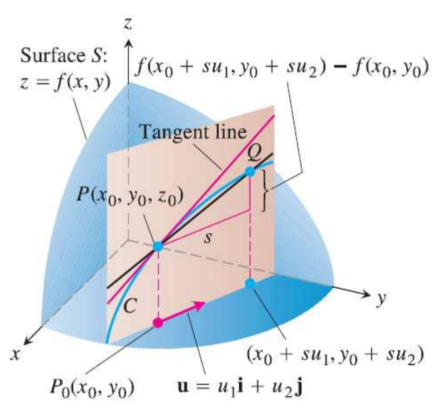

FIGURE 14.27 The slope of curve C at P_0 is $\lim_{Q \to P}$ slope (PQ); this is the directional derivative

$$\left(\frac{df}{ds}\right)_{\mathbf{u}, P_0} = (D_{\mathbf{u}} f)_{P_0}.$$

1.3 *Definition*

> The derivative of f at $P_0(x_0, y_0)$ in the direction of the unit vector $\vec{u} = u_1\vec{i} + u_2\vec{j}$ is the number
>
> ____________ = ______________________________________
>
> povided the limit exists.

1.4 The derivative of f at P_0 in the direction of $\vec{u}$ is also defined by ____________.

1.5 The partial defivatives $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ are the _________________________ of f at P_0 in the ______ and ______ directions.

Ex. 1 .... (`example1, p785`)

Using the definition, find the derivative of $f(x, y) = x^2 + xy$ at $P_0(1, 2)$ in the direction of the unit vector $\vec{u} = (1/\sqrt{2})\vec{i} + (1/\sqrt{2})\vec{j}$

sol:

2 Calculation and Gradients

2.1 *Definition*

> The gradient vector (____________) of $f(x, y)$ at a point $P_0(x_0, y_0)$ is the vector
>
> ________ = _______________

2.2 *Theorem 9: The Directional Derivative Is a Dot Product*

> If $f(x, y)$ is differentiable in an open region containing $P_0(x_0, y_0)$, then
>
> ________________________
>
> the dot product of ________________________ and ______.

✎ Ex. **2** .... (example2, p786)

Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\vec{v} = 3\vec{i} - 4\vec{j}$.

sol:

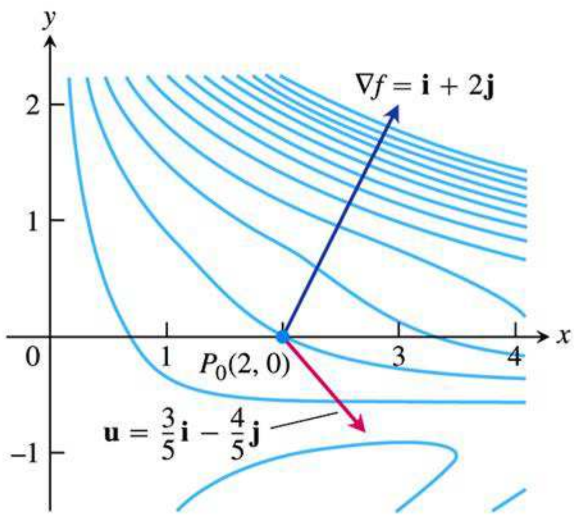

FIGURE 14.28 Picture ∇f as a vector in the domain of f. The figure shows a number of level curves of f. The rate at which f changes at $(2, 0)$ in the direction $\mathbf{u} = (3/5)\mathbf{i} - (4/5)\mathbf{j}$ is $\nabla f \cdot \mathbf{u} = -1$ (Example 2).

實習課練習 (EXERCISE 14.5)

5. Find the gradient of the function at the given point. $f(x,y) = \sqrt{2x+3y}$, $(-1,2)$

9. Find ∇f at the given point: $f(x,y,z) = e^{x+y}\cos z + (y+1)\sin^{-1} x$, $(0,0,\pi/6)$.

□ In Exercise 11-18, find the derivative of the function at P_0 in the direction of $\vec{u}$.

11. $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\vec{u} = 4\vec{i} + 3\vec{j}$.

13. $g(x,y) = \dfrac{x-y}{xy+2}$, $P_0(1,-1)$, $\vec{u} = 12\vec{i} + 5\vec{j}$.

15. $f(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$.

17. $g(x,y,z) = 3e^x \cos yz$, $P_0(0,0,0)$, $\vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$.

THOMAS' CALCULUS (12/E)

15.1 Double and Iterated Integrals over Rectangles

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1 Double Integrals

1.1 A rectangle with sides parallel to the coordinate axes: ______________________________.

1.2 *Definition: The Double Integrals*

> Let f be a function of two variables that is defined on a ____________________.
> If
>
> ____________________________
>
> exists, we say that f is ______________ on R. Moreover, __________________, called the double integral of f over R, is then given by
>
> $$\iint_R f(x,y)\ dA = \underline{\hspace{6cm}}$$

1.3 If $f(x) \geq 0$, ______________ represents the ____________________ under the curve $y = f(x)$ between a and b.

1.4 If $f(x,y) \geq 0$, ________________ represents the _____________________ under the surface $z = f(x,y)$ and above the rectangle R.

1.5 *Theorem: Fubini's Theorem (First Form)*

If $f(x, y)$ is continuous throughout the rectangular region $R : a \leq\leq b, c \leq y \leq d$, then

$V =$ ________________ $=$ ________________ $=$

$=$ ________________ ________________ .

The expression of the right, called an iterated or repeated integral.

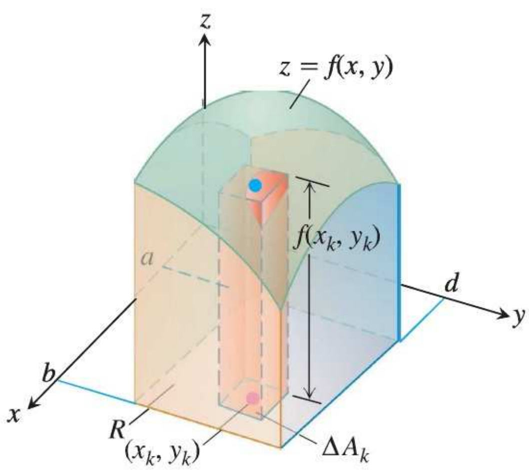
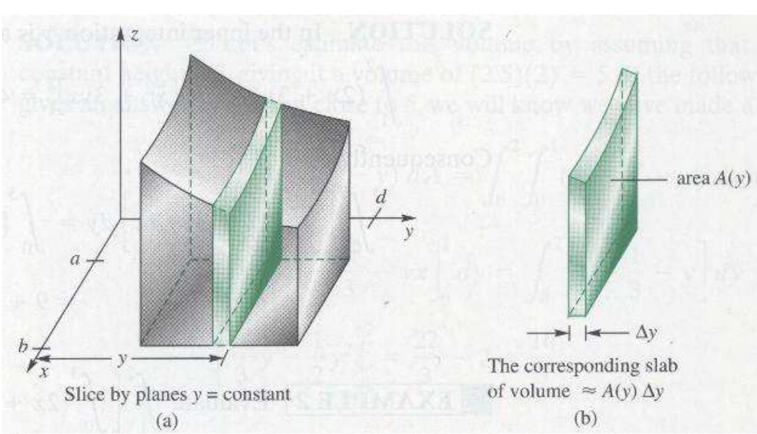
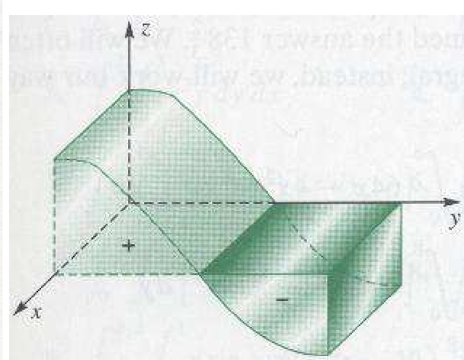


FIGURE 15.2 Approximating solids with rectangular boxes leads us to define the volumes of more general solids as double integrals. The volume of the solid shown here is the double integral of $f(x, y)$ over the base region R.

1.6 If $f(x, y)$ is ____________ on part of R, then $\iint_R f(x, y)\, dA$ gives the ________________ of the solid between the surface $z = f(x, y)$ and the rectangle R of the $xy-$plane. The actual volume of this slid is ________________ .





✎ Ex. **1** .... (example1, p839)

Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$ and $R: 0 \le x \le 2,\ -1 \le y \le 1$.

sol:

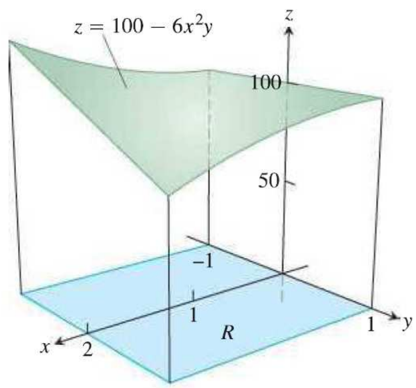


FIGURE 15.6 The double integral $\iint_R f(x, y)\,dA$ gives the volume under this surface over the rectangular region R (Example 1).

✎ Ex. **2** .... (example2, p840)

Find the volume of the region bounded above by the elliptical paraboloid $z = 10 + x^2 + 3y^2$ and below by the rectangle $R: 0 \le x \le 1, 0 \le y \le 2$.

sol:

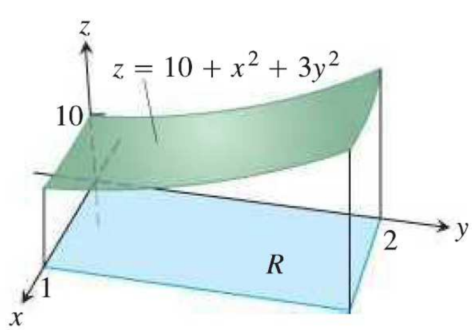


FIGURE 15.7 The double integral $\iint_R f(x, y)\,dA$ gives the volume under this surface over the rectangular region R (Example 2).

實習課練習 (EXERCISE 15.1)

4. $\int_0^1 \int_0^1 (1 - \frac{x^2 + y^2}{2})\ dx\ dy$

7. $\int_0^1 \int_0^1 \frac{y}{1 + xy}\ dx\ dy$

11. $\int_{-1}^2 \int_0^{\pi/2} y \sin x\ dx\ dy$

14. $\iint_R (\frac{\sqrt{x}}{y^2}) dA, \quad R : 0 \leq x \leq 4, 1 \leq y \leq 2.$

17. $\iint_R e^{x-y}, \quad R : 0 \leq x \leq \ln 2, 0 \leq y \leq 2.$

25. Find the volume of the region bounded above by the plane $z = 2 - x - y$ and below by the square $R : 0 \leq x \leq 1, 0 \leq y \leq 1$.

THOMAS' CALCULUS (12/E)

15.2 Double Integrals over General Regions

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1 Double Integrals over Bounded, Nonrectangular Regions

1.1 *Theorem: Fubini's Theorem (Stronger Form)*

Let $f(x, y)$ be continuous on a region R.

(a) If R is defined by $a \le x \le b, g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x,y)\ dA = \underline{\hspace{6cm}}.$$

(b) If R is defined by $c \le y \le d, h_1(y) \le x \le h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x,y)\ dA = \underline{\hspace{6cm}}.$$

Ex. 1 .... (example1, p843)

Find the volume of the prism whose base is the triangle in the xy-plane bounded by hte x-axis and the line $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$.

sol:

✎ Ex. **2** . . . . (`example2, p844`)

Calculate $\iint_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$, and the line $x = 1$.

sol:

2 Finding Limits of Integration

2.1 *Using Vertical Cross-sections*

When faced with evaluating $\iint_R f(x, y)\, dA$, integrating first with respect to y and then with respect to x, do the following:

(a) Sketch the ______________________ and label the __________________.

(b) Find the ___________ of integration. (c) Find the ___________ of integration.

2.2 *Using Horizontal Cross-sections*

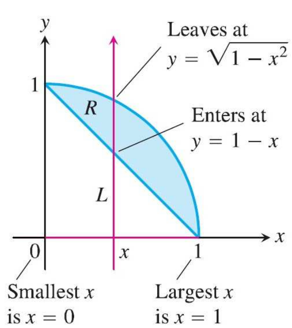

FIGURE 15.14 Finding the limits of integration when integrating first with respect to y and then with respect to x.

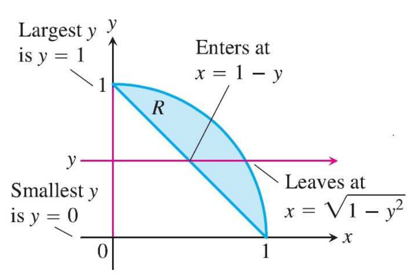

FIGURE 15.15 Finding the limits of integration when integrating first with respect to x and then with respect to y.

2.3 *Properties of Double Integrals*

If $f(x,y)$ and $g(x.y)$ are continuous on the bouneded region R, then

(a) Constant multiple: $\iint\limits_R kf(x,y)\ dA =$ ____________________.

(b) Sum and difference: $\iint\limits_R [f(x,y) \pm g(x,y)]\ dA =$ __________________________________.

(c) Additivity: $\iint\limits_R f(x,y)\ dA =$ ______________+________________+_____.

(d) Domination: If $f(x,y) \geq g(x,y)$, then __________________________________.

Ex. **3** .... (`example3, p846`)

$$\int_0^2 \int_{x^2}^{2x} (4x+2)\ dydx$$

sol:

Ex. **4** .... (`example4, p847`)

Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

sol:

實習課練習 (EXERCISE 15.2)

19. $\displaystyle\int_0^{\pi}\int_0^{x} x\sin y\ dydx.$

21. $\displaystyle\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\ dxdy.$

25. Integrate f over the given region: $f(x,y)=x/y$ over the region in the first quadrant bounded by the lines $y=x$, $y=2x$, $x=1$ and $x=2$.

36. $\displaystyle\int_0^{1}\int_{1-x}^{1-x^2}\ dydx.$

43. $\displaystyle\int_1^{e}\int_0^{\ln x} xy\ dydx.$

44. $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\ dydx.$

47. $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\ dydx.$

49. $\displaystyle\int_0^{1}\int_y^{1} x^2e^{xy}\ dxdy.$

60. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$, and the plane $z+y=3$.

THOMAS' CALCULUS (12/E)

15.3 Area by Double Integration

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1 Areas of Bounded Regions in the Plane

1.1 If we take $f(x, y) = 1$ in the definition of the double integral over a region R, the Riemann sums reduce to

$$S_n = \underline{\qquad\qquad\qquad\qquad} = \underline{\qquad\qquad}.$$

This is the sum of areas of all small rectangles in the partition of R.

1.2 *Definition*

The area of a closed, bounded plane region R is

$$A = \underline{\qquad\qquad\qquad\qquad\qquad}$$

1.3 The average value of f over R: $\underline{\qquad\qquad}$

Ex. 1 .... (example1, p850)

Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

sol:

✎ Ex. **2** .... (`example2, p851`)

Find the area of the region R bounded by $y = x^2$ and the line $y = x + 2$.

sol:

✎ Ex. **3** .... (`example3, p852`)

Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R : 0 \le x \le \pi, 0 \le y \le 1$.

sol:

實習課練習 (EXERCISE 15.3)

☐ In Exercise 1-12, express the region's area as an iterated double integral and evaluate the integral.

1. The coordinate axes and the line $x + y = 2$.

4. The parabola $x = y - y^2$ and the line $y = -x$.

14. $\int_0^3 \int_{-x}^{x(2-x)} dydx.$

18. $\int_0^2 \int_{x^2-4}^{0} dydx + \int_0^4 \int_0^{\sqrt{x}} dydx.$

21. Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \leq x \leq 2, 0 \leq y \leq 2$.

22. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \leq x \leq 2\ln 2, \ln 2 \leq y \leq 2\ln 2$

THOMAS' CALCULUS (12/E)

15.4 Double Integrals in Polar Form

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1 Integrals in Polar Coordinates

1.1 Integrals are sometimes easier to evaluate if we change to ____________.

1.2 Suppose that a function ________ is defined over a region R that is bounded by the rays ________ and ________, and by the continuous curves __________ and __________, where $0 \leq g_1(\theta) \leq g_2(\theta) \leq a$, and $\alpha \leq \theta \leq \beta$.

圖示如下:

1.3 (a) The polar rectangles that lie inside R, calling their areas ____________. Let ________ be any point in the polar rectangle whose area is ________.

(b) If f is continuous throughout R, the sum ________________ will approach a limit as we refine the grid to make ________________. The limit is called the double integral of f over R:

$$\lim_{n\to\infty} S_n = \underline{\qquad\qquad}$$

1.4 Write the sum S_n that expresses ΔA_k in terms of Δr and $\Delta\theta$.

圖示如下:

(a) Let the kth polar rectangle be ________. Let ______ be the average of the radii of the ________ and ____________ bounding the kth polar rectangle ΔA_k.

(b) The area of a wedge-shaped sector of a circle having radius r and angle θ: ______________.

(c) The area of the circular sectors:
Inner radius: __________________, Outer radius: __________________

(d) A_k =area of large sector − area of small sector

= ______________________________ = ________________________

(e) $S_n =$ ________________________. $\lim_{n\to\infty} S_n =$ ____________________.

1.5 The double integral of f over R is

$$\iint_R f(r,\theta)\ dA = \underline{\hspace{6cm}}$$

1.6 The area of a closed and bounded region R in the polar coordinate plane is

$$A = \underline{\hspace{4cm}}$$

1.7 *Changing Cartesian Integrals into Polar Integrals*

$$\iint_R f(x,y)\ dx\ dy = \underline{\hspace{6cm}},$$

where G denotes the same region of integration.

✎ **Ex. 1** .... (`example2, p855`)

Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

sol:

✐ Ex. **2** .... (example3, p856)

Evaluate $\iint\limits_R e^{x^2+y^2}\ dy\ dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

sol:

✐ Ex. **3** .... (example4, p856)

Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)\ dy\ dx$.

sol:

✐ Ex. **4** .... (example5, p856)

Find the volume of the solid region bounded above bby the paraboloid $z = 9-x^2-y^2$ and below by the unit circle in the xy-plane.

sol:

實習課練習 (EXERCISE 15.5)

9. $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\ dx.$

17. $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\ dy\ dx$

19. $\displaystyle\int_{0}^{\ln 2}\int_{0}^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$

20. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{-\sqrt{1-y^2}} \ln(x^2+y^2+1)\ dx\ dy$

27. Find the area of the region cut from the first quadrant by the curve $r = 2(2 - \sin 2\theta)^{1/2}$

28. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

105-2 小考 (1) 題目

2017/03/20, Calculus Quiz (1), §7.1 ∼ §7.5 (交回題目卷及答案卷)(每題 30 分)

1. (a) Show that if f has an interval I as domain and $f'(x)$ exists and is never zero on I, then $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$ where $a = f^{-1}(b)$. (b) Let $f(x) = x^3 - 3x^2 - 1, x \geq 2$. Find the value of df^{-1}/dx at $x = -1 = f(3)$ without finding a formula for $f^{-1}(x)$.

2. (a) Find dy/dx. (a) $y = \displaystyle\int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

3. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\displaystyle\int_2^4 x^{2x}(1+\ln x)\, dx$.

4. Find the limit. (a) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$. (b) $\displaystyle\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

105-2 小考 (2) 題目

2017/04/10, Calculus Quiz (2), §7.6 $\sim$ §8.3(交回題目卷及答案卷) (每小題 15 分，共 105 分)

1. $\displaystyle\int \frac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\,dx.$

2. (a) $\displaystyle\int \sqrt{\frac{x}{1-x^3}}\,dx.$ (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx.$

3. (a) Show that $\displaystyle\int x^n \sin x\,dx = -x^n \cos x + n\int x^{n-1}\cos x\,dx.$ (b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx.$

4. (a) $\displaystyle\int \sin^2 2x \cos^3 2x\,dx$ (b) $\displaystyle\int \sec^4 x \tan^2 x\,dx.$

105-2 小考 (3) 題目

2017/05/15, Calculus Quiz (3), §10.1 ∼ §10.8(交回題目卷及答案卷) (共 100 分)

1. (15 分) Find $\lim_{n\to\infty} a_n$ if the sequence $\{a_n = (\frac{x^n}{2n+1})^{1/n}, x > 0\}$ converges.

2. (30 分) Determine the convergence or divergence: (a)$\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$.

3. (30 分) Determine the convergence or divergence: (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

4. (25 分) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x+2)^n}{n}$.

105-2 小考 (4) 題目

2017/06/12, Calculus Quiz (4), §14.2 ~ §14.4, §15.1 ~ §15.3 (交回題目卷及答案卷) (共 100 分)

1. (20%) (a) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$. (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)}{x^2+y^2}$.

2. (20%) Find $\partial f/\partial x$ and $\partial f/\partial y$ if $f(x,y) = \displaystyle\sum_{n=1}^{\infty}(e^{xy})^n$ and $|e^{xy}| < 1$.

3. (30%) (a) Evaluate dw/dt at $t=1$: $w = 2ye^x - \ln z$, $x = \ln(t^2+1)$, $y = \tan^{-1} t$, $z = e^t$.

   (b) Find $\partial z/\partial x$ and $\partial z/\partial y$ at (0,0,0) if $x^3 + z^2 + ye^{xz} + z\sin xy = 0$.

4. (30%) (a) $\displaystyle\int_0^2 \int_x^2 2y^2 \sin xy \; dy \; dx$. (b) $\displaystyle\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}} \; dx \; dy$.

國立臺北大學 105 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分	開課班別： 通訊 1/電機 1/智財學程				命題教授: 吳漢銘
考試日期：4 月 17 日 (一) 10:10～11:40	※准帶項目打「O」，否則打「×」				1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 80 份	計算機	課本	筆記	電子辭典	紙本字典
備註：注意事項要看!!	×	×	×	×	×

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 120 分。

1. (10 分) Find dy/dx $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$.

2. (10 分) $\lim_{x \to e^+} (\ln x)^{1/(x-e)}$.

3. (10 分) $\int \sqrt{\frac{x}{1-x^3}}\, dx$.

4. (10 分) $\int \frac{x^3}{\sqrt{x^2+4}}\, dx$.

5. (10 分) Derive the integration by parts formula.

6. (10 分) What is the definition of the improper integrals (type I and type II)?

7. (10 分) Using trigonometric substitutions to evaluate the integrals: $\int \frac{8\, dx}{(4x^2+1)^2}$.

8. (10 分) Use partial fractions to evaluate $\int \frac{2x+2}{(x^2+1)(x-1)^3}\, dx$.

9. (10 分) Evaluating improper integrals: $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$.

10. (10 分) Testing for convergence: $\int_1^\infty \frac{1}{e^x - 2^x}\, dx$.

11. (20 分) Find the values of p for which the integral $\int_2^\infty \frac{dx}{x(\ln x)^p}$ converges.

國立臺北大學 105 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1/電機 1/智財學程　　命題教授: 吳漢銘

考試日期：6 月 19 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
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備註：注意事項要看!!	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 110 分。

1. (10 分) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}(x-2)^n}{3n}$.

2. (10 分) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$.

3. (20 分) (a) Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point. $(3, \pi/4)$, $(-3, \pi/4)$, $(3, -\pi/4)$, $(-3, -\pi/4)$. (b) Replace the Cartesian equation $(x+2)^2 + (y-5)^2 = 16$ by equivalent polar equations.

4. (20 分) Let $\vec{v} = -\vec{i} + \vec{j}$, $\vec{u} = \sqrt{2}\,\vec{i} + \sqrt{3}\,\vec{j} + 2\vec{k}$, find Find (a) $\vec{v} \cdot \vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$; (b) the cosine of the angle between $\vec{v}$ and $\vec{u}$. (c) the scalar component of $\vec{u}$ in the direction of $\vec{v}$. (d) the vector $\text{proj}_{\vec{v}}\vec{u}$.

5. (10 分) Let $f(x,y) = \begin{cases} \sqrt{y^3}, & y \geq 0 \\ -y^2, & y < 0. \end{cases}$ Using the definition of the partial derivative of $f(x,y)$ to find $f_y(x,0)$.

6. (20 分) (a) Suppose that $F(x,y)$ is differentiable and that the equation $F(x,y) = 0$ defines y as a differentiable function of x. Show that, at any point where $F_y \neq 0$, $dy/dx = -F_x/F_y$.
   (b) Suppose that the equation $F(x,y,z) = xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defines the variable z implicitly as a function $z = f(x,y)$. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(1, \ln 2, \ln 3)$.

7. (10 分) Find the volume of the solid in the first octant (八分圓) bounded by the coordinate planes, the plane $x = 3$, and the parabolic (拋物線的) cylinder (圓柱) $z = 4 - y^2$.

8. (加分題 10 分) (a) 本課程助教姓名: ________________。(b) 助教的研究室在哪一間或助教都在哪一間教室回答同學們的問題: ________________。

2018/03/26, Calculus Quiz (1), §7.2 ∼ §7.6 (交回題目卷及答案卷)(每題 30 分)

1. (a) Find dy/dx. (a) $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

2. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\, dx$.

3. Find the limit. (a) $\lim_{x\to 1^+} (\dfrac{1}{x-1} - \dfrac{1}{\ln x})$. (b) $\lim_{x\to e^+} (\ln x)^{1/(x-e)}$.

4. Evaluate the integrals. (a) $\int \dfrac{1}{(\sin^{-1} x)\sqrt{1-x^2}}\, dx$. (b) $\int \dfrac{dx}{(x+1)\sqrt{x^2+2x}}$.

. . . .

2018/03/26, Calculus Quiz (1), §7.2 ∼ §7.6 (交回題目卷及答案卷)(每題 30 分)

1. (a) Find dy/dx. (a) $y = \int_{x^2/2}^{x^2} \log_5 \sqrt{t}\, dt$. (b) $y = \sqrt[3]{\dfrac{x^2(x-1)}{x^2+1}}$.

2. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\, dx$.

3. Find the limit. (a) $\lim_{x\to 1^+} (\dfrac{1}{x-1} - \dfrac{1}{\ln x})$. (b) $\lim_{x\to e^+} (\ln x)^{1/(x-e)}$.

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2018/06/04, Calculus Quiz (3), §8.7 ∼ §10.5 (交回題目卷及答案卷)(每題 30 分)

1. Determine the convergence of the series $\sum_{n=2}^{\infty}(-1)^n \frac{\ln n}{n - \ln n}$. (converges absolutely, converge conditionally or diverges)

2. Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?
(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$, (b) $\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$.

3. Find the Taylor series generated by $f = 2^x$ at $x = 1$.

4. Let $f(x,y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

5. Assuming that the equation $xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defined y as a differentiable function of x, find the value of dy/dx at the point $(1, \ln 2, \ln 3)$.

. . . .

2018/06/04, Calculus Quiz (3), §8.7 ∼ §10.5 (交回題目卷及答案卷)(每題 30 分)

1. Determine the convergence of the series $\sum_{n=2}^{\infty}(-1)^n \frac{\ln n}{n - \ln n}$. (converges absolutely, converge conditionally or diverges)

2. Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?
(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$, (b) $\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$.

3. Find the Taylor series generated by $f = 2^x$ at $x = 1$.

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5. Assuming that the equation $xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$ defined y as a differentiable function of x, find the value of dy/dx at the point $(1, \ln 2, \ln 3)$.

國立臺北大學 106 學年度第 2 學期 期中 考試命題紙

考試科目：微積分	開課班別：通訊 1/電機 1/智財學程				命題教授：吳漢銘
考試日期：4 月 16 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」				1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 77 份	計算機	課本	筆記	電子辭典	紙本字典
備註：注意事項要看!!	×	×	×	×	×

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 每題 10 分，總分共 100 分。

1. (a) Show that the number e can be calculated as the limit. (b) Evaluate $\int_2^4 x^{2x}(1+\ln x)\,dx$.

2. Find the limit. (a) $\lim_{x\to 1^+}(\frac{1}{x-1}-\frac{1}{\ln x})$. (b) $\lim_{x\to e^+}(\ln x)^{1/(x-e)}$.

3. $\int \cos\sqrt{x}\,dx$

4. $\int \sqrt{x}e^{\sqrt{x}}\,dx$

5. $\int \sin^3 x\cos^3 x\,dx$

6. $\int_{-\pi}^{\pi}(1-\cos^2 t)^{3/2}\,dt$

7. $\int \frac{x^2}{(x^2-1)^{5/2}}\,dx$

8. $\int \frac{x}{\sqrt{1+x^4}}\,dx$

9. $\int \frac{9x^3-3x+1}{x^3-x^2}\,dx$

10. $\int \frac{1}{x^6(x^5+4)}\,dx$ (hint: let $u=x^5$)

注意： 1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。

2、考試時不得有交談、窺視、夾帶、抄襲、傳遞、代考或其它作弊等舞弊行為，考畢務必交卷，不得攜卷出場，違者依考場規則議處。

國立臺北大學 106 學年度第 2 學期 期末 考試命題紙

考試科目：微積分　開課班別：通訊 1/電機 1/智財學程　命題教授：吳漢銘

考試日期：6 月 25 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
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1. (10 分) Determine the convergence of the series (converge absolutely, converge conditionally, or diverge). $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$

2. (10 分) (a) Find the series' radius and interval of convergence. Then identify the values of x for which the series converges (b) absolutely and (c) conditionally. $\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^{2n+1}}{2n+1}$.

3. (10 分) Find the Taylor series generated by $f(x) = 2^x$ at $a = 1$.

4. (10 分) Assuming that the equation $2xy + e^{x+y} - 2 = 0$ define y as a differentiable function of x, find the value of dy/dx at point $P(0, \ln 2)$.

5. (10 分) Assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

6. (15 分) Write an iterated integral for $\iint_R dA$ over the described region $R = \{$bounded by $y = x^2$ and $y = x + 2\}$ using (a) vertical cross-sections, (b) horizontal cross-sections.

7. (15 分) Evaluate the integral: $\int_0^2 \int^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$.

8. (15 分) Find the volume of the solid in the first octant (八分圓) bounded by the coordinate planes, the plane $x = 3$, and the parabolic (拋物線的) cylinder (圓柱) $z = 4 - y^2$.

9. (15 分) Find the average value of $f(x, y) = xy$ over the regions: the quarter circle $x^2 + y^2 \leq 1$ in the first quadrant.

2019/03/18, Calculus Quiz (1), §7.5 ~ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$, (b) $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$, (c) $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

2. (10%) 導出分部積分 (Integration by parts) 的公式。

3. (30%) (a) $\int \frac{dx}{x\sqrt{25x^2-2}}$, (b) $\int_0^{\ln\sqrt{3}} \frac{e^x\ dx}{1+e^{2x}}$, (c) $\int \frac{dy}{(\sin^{-1}y)\sqrt{1-y^2}}$

4. (30%) (a) $\int e^\theta \sin\theta\ d\theta$, (b) $\int \ln(x+x^2)\ dx$, (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

. . . .

2019/03/18, Calculus Quiz (1), §7.5 ~ §8.1 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (30%) (a) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$, (b) $\lim_{x\to 0}\frac{(e^x-1)^2}{x\sin x}$, (c) $\lim_{x\to\infty}(\frac{x^2+1}{x+2})^{1/x}$.

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4. (30%) (a) $\int e^\theta \sin\theta\ d\theta$, (b) $\int \ln(x+x^2)\ dx$, (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$

2019/05/06, Calculus Quiz (2), §10.4 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (20%) Use the Comparison Test or the Limit Comparison Test to determine if each series converges or diverges. (a) $\sum_{n=1}^{\infty}(\frac{2n+3}{5n+4})^n$. (b) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$.

2. (20%) Determine the convergence of the series (conditionally, absolutely, or diverges)
   (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$. (b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+\sqrt{n+1}}$.

3. (30%) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? (a) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$. (b) $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$.

4. (10%) Find the Maclaurin series of xe^x.

5. (20%) Find the Taylor series and the Taylor polynomials generated by $f(x)=\sin x$ at $x=0$.

....

2019/05/06, Calculus Quiz (2), §10.4 ∼ §10.8 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (20%) Use the Comparison Test or the Limit Comparison Test to determine if each series converges or diverges. (a) $\sum_{n=1}^{\infty}(\frac{2n+3}{5n+4})^n$. (b) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$.

2. (20%) Determine the convergence of the series (conditionally, absolutely, or diverges)
   (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$. (b) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+\sqrt{n+1}}$.

3. (30%) Find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally? (a) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n x^n$. (b) $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$.

4. (10%) Find the Maclaurin series of xe^x.

5. (20%) Find the Taylor series and the Taylor polynomials generated by $f(x)=\sin x$ at $x=0$.

2019/05/27, Calculus Quiz (3), §14.4 ∼ §15.2 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (10%) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$.

2. (20%) Let $f(x,y) = \begin{cases} y\sqrt{x}, & x \geq 0, y \in R \\ yx^2, & x < 0, y \in R. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

3. (30%) (a) Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$. (b) Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u,v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u \sin v$, $(u,v) = (1.3, \pi/6)$.

4. (20%) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

5. (20%) (a) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\ dydx$. (b) $\displaystyle\int_0^1\int_y^1 x^2e^{xy}\ dxdy$.

....

2019/05/27, Calculus Quiz (3), §14.4 ∼ §15.2 (可用鉛筆、需計算過程、交回題目卷及答案卷)

1. (10%) $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$.

2. (20%) Let $f(x,y) = \begin{cases} y\sqrt{x}, & x \geq 0, y \in R \\ yx^2, & x < 0, y \in R. \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

3. (30%) (a) Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$. (b) Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u,v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u \sin v$, $(u,v) = (1.3, \pi/6)$.

4. (20%) Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

5. (20%) (a) $\displaystyle\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\ dydx$. (b) $\displaystyle\int_0^1\int_y^1 x^2e^{xy}\ dxdy$.

國立臺北大學 107 學年度第 2 學期 期中 考試命題紙

考試科目： 微積分 　　開課班別： 資工 1 　　命題教授: 吳漢銘

考試日期：4 月 15 日 (一) 10:10～11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 105 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!! (§7.5～§10.3)	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號 (含小題) 需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 114 分。

1. (16 分) (a) $\lim_{x\infty}(\frac{x^2+1}{x+2})^{1/x}$. (b) $\int \ln(x+x^2)\ dx$,

2. (16 分) (a) $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\ d\theta$. (b) $\int_0^{\pi/2} \theta\sqrt{1-\cos 2\theta}\ d\theta$.

3. (16 分) (a) $\int \frac{v^2}{(1-v^2)^{5/2}}\ dv$. (b) $\int \frac{x\ dx}{\sqrt{1+x^4}}$.

4. (16 分) (a) $\int \frac{8x^2+8x+2}{(4x^2+1)^2}\ dx$. (b) $\int \frac{e^t\ dt}{e^{2t}+3e^t+2}$.

5. (16 分) (a) $\int_{-\infty}^{\infty} \frac{2x\ dx}{(x^2+1)^2}$. (b) $\int_2^4 \frac{dt}{t\sqrt{t^2-4}}$.

6. (8 分) Does the integral $\int_1^{\infty} \frac{1}{\sqrt{e^x-x}}$ converge?

7. (10 分) Discuss (with proofs) the convergence of the p-series.

8. (16 分) Determining convergence or divergence. (a) $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$. (b) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$.

國立臺北大學 107 學年度第 2 學期 期末 考試命題紙

考試科目：微積分		開課班別：資工 1			命題教授：吳漢銘
考試日期：06 月 17 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」				1. 需加發計算紙或答案紙請在試題內封袋備註。2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 100 份	計算機	課本	筆記	電子辭典	紙本字典
備註：注意事項要看!!	×	×	×	×	×

A 卷

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 同時交回答案卷、題目卷、計算紙。(7) 總分共 100 分。(8) 請於答案卷的記分欄下方，填上卷別「A」或「B」。

1. (10 分) Given the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}+\sqrt{n+1}}$, determine the convergence of the series (conditionally, absolutely, or diverges).

2. (10 分) Given the series $\sum_{n=0}^{\infty}(\sqrt{n+1}-\sqrt{n})(x-3)^n$, find the series's radius and interval of convergence. For what values of x does the series converge absolutely, or conditionally?

3. (10 分) Find the volume of the wedgelike solid that lies beneath the surface $z=16-x^2-y^2$ and above the region R bounded by the curve $y=2\sqrt{x}$, the line $y=4x-2$, and the x-axis.

4. (10 分) $\int_0^{\pi/6}\int_{\sin x}^{1/2} xy^2\ dydx$.

5. (10 分) Replace the following Cartesian equation by equivalent polar equations.
   (a) $xy=2$. (b) $x^2+xy+y^2=1$.

6. (20 分) Graph the curve $r^2=4\cos 2\theta$.

7. (20 分) Find the area of the region inside the circle $r=3a\cos\theta$ and outside the cardioid $r=a(1+\cos\theta),\ a>0$.

8. (10 分) Express the region's area as an iterated double integral and evaluate the integral: The parabola $x=y-y^2$ and the line $y=-x$.

注意： 1、考試求公平及公正，請同學務必自律，維護學校與學生之榮譽。

2、考試時不得有交談、窺視、夾帶、抄襲、傳遞、代考或其它作弊等舞弊行為，考畢務必交卷，不得攜卷出場，違者依考場規則議處。

微積分會考−考古題

科　　目：微積分

考試時間：100 分鐘

※ 注意：

（一）本試題共有單選題 20 題(選項：五選一)，填空題 10 題。

（二）選擇題請選出一個正確或最適當的答案，用 2B 鉛筆依題號於試卡上清楚劃記，複選作答者，該題不予計分。

（三）填空題請將正確答案依題號填於答案卷空格中。

（四）本試題不可使用電子計算器。

（五）本試題不得攜帶出考場。

一、選擇題：

1. Calculate $\lim_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}=$

   (A) ∞ (B) $-\infty$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{1}{\sqrt{2}}$

2. Choose an equation from the following that expresses the fact that a function f is continuous at the number 6.

   (A) $\lim_{x\to 0} f(x)=6$ (B) $\lim_{x\to 0} f(x)=f(6)$ (C) $\lim_{x\to 6} f(x)=6$

   (D) $\lim_{x\to 6} f(x)=f(0)$ (E) $\lim_{x\to 6} f(x)=f(6)$

3. Let $f(x)=\begin{cases} x+7 & , x<3 \\ -x^2+9x-7, & x\geq 3\end{cases}$. Which one of the following statements is TRUE ?

   (A) $\lim_{x\to 3^-} f(x)=11$ (B) $\lim_{x\to 3^+} f(x)=11$ (C) $\lim_{x\to 3} f(x)=11$

   (D) $f(3)=10$ (E) f is continuous at 3

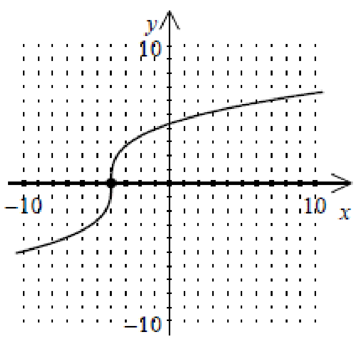
4. Determine where the function is differentiable.

   (A) $x=-4$ (B) $y=0$

   (C) $y\neq 5$

   (D) All x except $x=-4$

   (E) The function is differentiable everywhere.



微積分會考−考古題

5. If $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=a$, $a\in ¡$, which one of the following statements is FALSE?
   (A) $f(c)=0$ (B) $f'(c)=a$ (C) f is differentiable at c
   (B) f is continuous at c (E) $\lim_{x\to c}f(x)=f(c)$

6. Find the equation of the tangent line to the curve $f(x)=x^3-x^2+6$ at the point (1, 6).
   (A) $y=2x+4$ (B) $y=x+5$ (C) $2y=x+11$ (D) $y=-x+7$
   (E) $y=-2x+8$

7. Evaluate $\frac{d}{dx}\int_0^1 e^{x^2}dx=$
   (A) e^{x^2} (B) $2xe^{x^2}$ (C) $e-1$ (D) $2e$ (E) 0

8. The annual sales x of a new consumer product can be modeled by $x=\frac{15,000t^2}{36+t^2}$, where t is the number of years since the introduction of the product. Find the time at which the sales are increasing most rapidly.
   (A) $2\sqrt{3}$ years (B) $3\sqrt{2}$ years (C) $3\sqrt{3}$ years
   (D) $3\sqrt{5}$ years (E) $2\sqrt{6}$ years

9. Let $f(x)=x\ln x$, $x>0$. Then the minimum value of $f(x)$ is:
   (A) 0 (B) 1 (c) e (D) $-e$ (E) $-\frac{1}{e}$

10. Evaluate $\int_2^5[x]dx$, where $[x]$ is the largest integral less than or equal to x.
   (A) 7 (B) 8 (C) 9 (D) 10 (E) 10.5

11. The indefinite integral $\int x^2\ln 5x dx$
   (A) $\frac{x^2}{2}(2\ln 5x-1)+c$ (B) $\frac{x^3}{3}(3\ln 5x+1)+c$ (C) $\frac{x^2}{4}(2\ln 5x+1)+c$
   (D) $\frac{x^3}{9}(3\ln 5x-1)+c$ (E) $\frac{x^2}{15}(2\ln 5x-1)+c$

12. The indefinite integral of $\int\frac{1}{x^2-1}dx$ is :
   (A) $\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+c$ (B) $\ln|x^2-1|+c$ (C) $\frac{1}{x}\ln|x^2-1|+c$
   (D) $\tan^{-1}x+c$ (E) $\sec^{-1}x+c$

微積分會考−考古題

13. Find the particular solution, $y = f(x)$, that satisfies the conditions

$f''(x) = \frac{6}{\sqrt{x}} + 3$, $f'(1) = 12$, $f(4) = 56$.

(A) $f(x) = 12x\sqrt{x} + \frac{3}{2}x^2 + 6x - 88$ (B) $f(x) = 3x\sqrt{x} + \frac{3}{2}x^2 + 7x - 20$

(C) $f(x) = 8x\sqrt{x} + \frac{3}{2}x^2 - 3x - 20$ (D) $f(x) = 12\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$

(E) $f(x) = 3\sqrt[3]{x} + \frac{3}{2}x^2 + 6x - 8$

14. The definite integral of $\int_{-1}^{1}(e^{-x} + 1)dx$ is:

(A) $-e^{-x} + x + c$ (B) $e - e^{-1} + 2$ (C) $e + e^{-1}$ (D) 2 (E) 0

15. Find the area of the region bounded by the graphs $f(y) = y^2 + 1$ and $g(y) = 4 - 2y$.

(A) $\frac{31}{3}$ (B) $\frac{23}{4}$ (C) $\frac{31}{4}$ (D) $\frac{32}{3}$ (E) $\frac{22}{3}$

16. If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

17. The indefinite integral of $\int e^{-(x-\theta)}dx$ is

(A) $-\frac{1}{x-\theta}e^{-(x-\theta)} + c$ (B) $-\frac{1}{\theta}e^{-(x-\theta)} + c$ (C) $-\frac{1}{x}e^{-(x-\theta)} + c$

(D) $-\theta e^{-(x-\theta)} + c$ (E) $-e^{-(x-\theta)} + c$

18. Find the limit of $\lim_{x\to\infty}(1 + \frac{2}{x})^{5x}$.

(A) 1 (B) e^5 (C) e^{10} (D) ∞ (E) 0

19. Let $f(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$. Which one of the following points is NOT a critical point of f ?

(A) $(\sqrt{3}, -1)$ (B) $(-\sqrt{3}, 1)$ (C) $(-\sqrt{3}, -1)$ (D) $(0, 2)$ (E) $(0, 0)$

20. Evaluate the value of $\int_0^2\int_0^x xydydx$?

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) 1 (D) 2 (D) 4

微積分會考－考古題

二、填空題：

1. Let $f(x)=\dfrac{x^2+x-6}{|x-2|}$, $x\neq 2$. Then $\lim\limits_{x\to 2^-} f(x)=$__________

2. $\lim\limits_{x\to\infty}(\sqrt{x^2+x+1}-\sqrt{x^2-x+1})=$__________

3. Suppose that $f'(a)=2$, then $\lim\limits_{h\to 0}\dfrac{f(a+2h)-(a-h)}{h}=$__________

4. $\lim\limits_{x\to 0^+} x\ln x=$__________

5. Evaluate $\int\dfrac{x}{\sqrt{x^2+1}}dx=$__________

6. The improper integral of $\int_1^\infty \dfrac{\ln x}{x^2}dx=$__________

7. The area of the region bounded by the curves $y=e^x$ and $y=e^{-x}$ between $x=0$ and $x=\ln 5$ is: __________

8. If $F(X)=\int_0^{\ln x}\sqrt{e^t+1}dt$, then $F'(x)=$__________

9. Let $f(x,y)=y\ln x+2xy^2$. The directional derivative of f at $(1,-2)$ is: __________

10. Evaluate the improper integral $\int_e^\infty \dfrac{1}{x(\ln x)^3}dx$__________.

微積分會考-考古題

一、選擇題答案

1	2	3	4	5
C	E	B	D	A
6	7	8	9	10
B	E	A	E	C
11	12	13	14	15
D	A	C	B	D
16	17	18	19	20
A	E	C	B	D

二、填空題答案

1	2	3	4	5
-5	1	6	0	$\sqrt{x^2+1}+C$
6	7	8	9	10
1	$\frac{16}{5}$	$\frac{\sqrt{x+1}}{x}$	$[6,\ -8]$	$\frac{1}{2}$