1. (a) ① Let $n-1$ points $\{x_1, x_2, \cdots x_{n-1}\}$ in $[a,b]$, satisfying $a = x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n = b$

② A partition of $[a,b]$: $P = \{x_0, x_1 \cdots x_{n-1}, x_n\}$

③ The k^{th} sub-interval of P is $[x_{k-1}, x_k]$

(10%) ④ The norm of a partition P, $\|P\|$, the largest of all sub-interval widths.

⑤ A Riemann sum of f on interval $[a,b]$: $\boxed{S_P = \sum_{k=1}^{n} f(C_k)\Delta x_k}$ for every $C_k \in [x_{k-1}, x_k]$, $k = 1, \dots n$.

(b) part I: If f is continuous on $[a,b]$ then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a,b]$, and diffeuetiable on (a,b)

(10%) and its derivative is $f(x)$; $F'(x) = \frac{d}{dx}\int_a^x f(x)\,dx = f(x)$

part II: If f is continuous at every point of $[a,b]$, and F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

(5%)

2. let $p = \{-1, -1+\frac{1}{n}, -1+\frac{2}{n}, \cdots, -1+\frac{n-1}{n}, 0\}$, $C_k \in [x_{k-1}, x_k]$, $\Delta x_k = \frac{1}{n}$

$$S_p = \sum_{k=1}^{n} f(C_k)\Delta x_k = \sum_{k=1}^{n} f(-1+\frac{k}{n})\cdot\frac{1}{n} = \sum_{k=1}^{n}\left[(\frac{k}{n}-1) - (\frac{k}{n}-1)^2\right]\cdot\frac{1}{n} = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{k}{n} - 1 - \frac{k^2}{n^2} + \frac{2k}{n} - 1\right]$$

$$= -\frac{1}{n^3}\sum_{k=1}^{n} k^2 + \frac{3}{n^2}\sum_{k=1}^{n} k - \frac{2}{n}\sum_{k=1}^{n} 1 = -\frac{1}{n^{\cancel{3}2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{3}{n^2}\cdot\frac{n(n+1)}{2} - \frac{2}{\cancel{n}}\cdot\cancel{n}$$

(25%)

$$= -\frac{2n^2+3n+1}{6n^2} + \frac{3(n-1)}{2n} - 2 = \frac{-1}{3} + \frac{3n+1}{6n^2} + \frac{3}{2} - \frac{9n}{6n^2} - 2$$

$$= \frac{-5}{6} + \frac{-6n+1}{6n^2} = \frac{-5n^2-6n+1}{6n^2}$$

$$\int_{-1}^{0}(x - x^2)\,dx = \lim_{n\to\infty} S_p = \lim_{n\to\infty}\left(\frac{-5}{6} + \frac{-6n+1}{6n^2}\right) = \frac{-5}{6} \#$$

0

3. (a) $$\frac{dy}{dx} = \frac{d}{dx}\int_0^{\sin 2x}\frac{dt}{\sqrt{1-t^2}} = \frac{1}{\sqrt{1-\sin^2 2x}}\cdot\cos 2x\cdot 2 = \frac{2\cos 2x}{\sqrt{1-\sin^2 2x}} = \frac{2\cos 2x}{|\cos 2x|} \#$$

(10%)

(b) let $u = x^2+1$, $du = 2x\,dx$, $x^2 = u-1$.

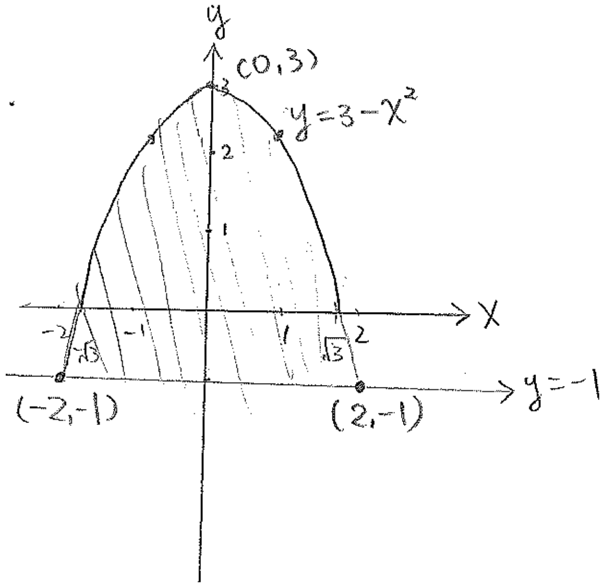
$$\int x^3\sqrt{x^2+1}\,dx = \frac{1}{2}\int x^2\sqrt{x^2+1}\cdot(2x)\,dx = \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int (u^{\frac{3}{2}} - u^{\frac{1}{2}})\,du$$

(15%)

$$= \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right] + C = \left(\frac{1}{5}u^{\frac{5}{2}} - \frac{1}{3}u^{\frac{3}{2}}\right) + C = \frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}} + C \#$$

4.

(a).



(10%)

by x:

$$\text{area} = \int_{-2}^{2} [(3-x^2)-(-1)]\,dx$$

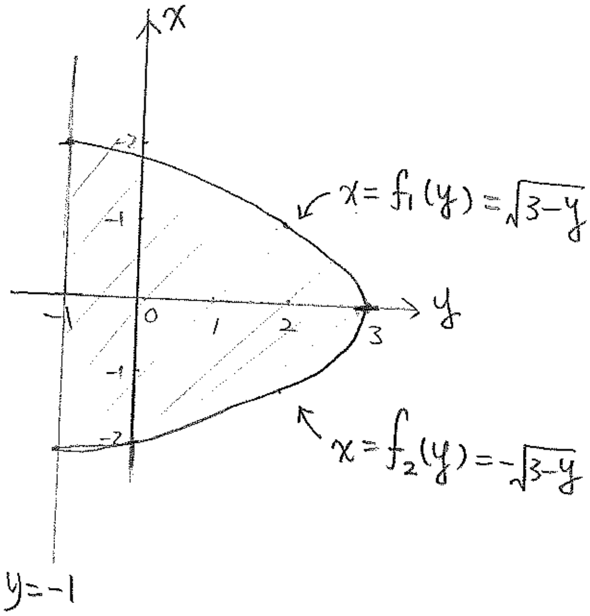
$$= \int_{-2}^{2} (4-x^2)\,dx$$

$$= 4x - \frac{1}{3}x^3 \Big|_{-2}^{2}$$

$$= \left(8-\frac{8}{3}\right) - \left(-8+\frac{8}{3}\right)$$

$$= 16 - \frac{16}{3} = \frac{32}{3} \#$$

(b)



(15%)

by y:

$$\text{area} = \int_{-1}^{3} (\sqrt{3-y}) - (-\sqrt{3-y})\,dy$$

$$= \int_{-1}^{3} 2\sqrt{3-y}\,dy, \quad \text{let } \begin{matrix} u=3-y \\ du=-dy \end{matrix}$$

$$= 2\int_{4}^{0} \sqrt{u}\cdot(-du)$$

$$= 2\int_{0}^{4} u^{\frac{1}{2}}\,du$$

$$= 2\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_{0}^{4}$$

$$= \frac{4}{3}\cdot(8-0) = \frac{32}{3} \#$$