

1. (a) Instantaneous speed at t_0 is average speed during $[t_0, t_0+h]$ as $h \rightarrow 0$ (10%)

(b) Suppose that $h(x) \leq f(x) \leq g(x)$ for all x in some open interval containing c , except possibly at $x=c$ itself. Suppose also that $\lim_{x \rightarrow c} h(x) = L$ and $\lim_{x \rightarrow c} g(x) = L$, then $\lim_{x \rightarrow c} f(x) = L$. (10%)

(c) The function f is continuous at point c if:

1. $f(c)$ is defined.

2. $\lim_{x \rightarrow c} f(x)$ exists

3. $\lim_{x \rightarrow c} f(x) = f(c)$. (10%)

(d) if for every number $\epsilon > 0$, there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \epsilon$, whenever $0 < |x - c| < \delta$ (10%)

2. let $\epsilon > 0$ be given, $\forall x$ $|x-1| < \delta$.

if $x < 1$, $|f(x) - 2| = |2 - 2x| = 2|1-x| < 2\delta < 6\delta$

if $x \geq 1$, $|f(x) - 2| = |6x - 6| = 6|x-1| < 6\delta \Rightarrow$

For $\epsilon = 6\delta$, we have $|f(x) - 2| < \epsilon$ which prove $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = 2$ (20%)

3. (a) $\lim_{x \rightarrow 0^+} (4(g(x)))^{\frac{1}{3}} = 2 \Rightarrow \lim_{x \rightarrow 0^+} (4g(x)) = 8 \Rightarrow \lim_{x \rightarrow 0^+} g(x) = 8/4 = 2$ (10%)

(b) $\lim_{x \rightarrow \sqrt{5}} \frac{1}{x+g(x)} = 2 \Rightarrow \lim_{x \rightarrow \sqrt{5}} (x+g(x)) = \frac{1}{2} \Rightarrow \lim_{x \rightarrow \sqrt{5}} x + \lim_{x \rightarrow \sqrt{5}} g(x) = \frac{1}{2}$

$\Rightarrow \lim_{x \rightarrow \sqrt{5}} g(x) = \frac{1}{2} - \sqrt{5}$ (10%)

4. (a) $\lim_{\theta \rightarrow 3^+} \frac{\lfloor 2\theta \rfloor}{\theta} = \frac{\lim_{\theta \rightarrow 3^+} \lfloor 2\theta \rfloor}{\lim_{\theta \rightarrow 3^+} \theta} = \frac{6}{3} = 2$ (10%)

(b) $\lim_{\theta \rightarrow 4^-} (\theta - \lfloor 2\theta \rfloor) = \lim_{\theta \rightarrow 4^-} \theta - \lim_{\theta \rightarrow 4^-} \lfloor 2\theta \rfloor = 4 - 7 = -3$ (10%)