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未依precise def:(0%)
左.右ε不同:(5%)

1. let $\varepsilon>0$ be given, $\forall x \ |x-1|<\delta$

(10%)

if $x<1$, $|f(x)-2| = |2-2x| = 2|x-1| < 2\delta$

if $x\geq 1$, $|f(x)-2| = |6x-6| = 6|x-1| < 6\delta$

take $\delta = \varepsilon/6$, we have $\begin{cases} |f(x)-2| < 2\delta = \varepsilon/3 < \varepsilon, & x<1 \\ |f(x)-2| < 6\delta = \varepsilon, & x\geq 1 \end{cases}$,

which prove $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = 2$.

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2. state: If f is a differentiable function at x_0, then it is continuous at x_0. (2)

(10%)

proof:

$$\lim_{h\to 0} f(x_0+h) = \lim_{h\to 0}\left(f(x_0+h) - f(x_0) - f(x_0)\right)$$ (8)

$$= \lim_{h\to 0}[f(x_0)] + \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}\cdot\lim_{h\to 0} h$$

$$= f(x_0) + f'(x_0)\cdot 0 = f(x_0)$$ ∗ 得証$f(x)$在x_0連續

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3. (a) $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (7)

(14%)

(b) 若$f(x)$在$[a,b]$間連續, 且u滿足$f(a)<u<f(b)$或$f(b)<u<f(a)$

則存在$f(c)\in(a,b)$使得$f(c)=u$. (7), 若未提反連續(3)

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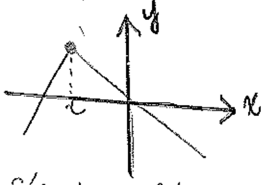
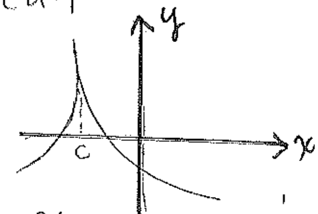
4. (10%)

(a) corner



$f'(c^+) \neq f'(c^-)$

(b) cusp



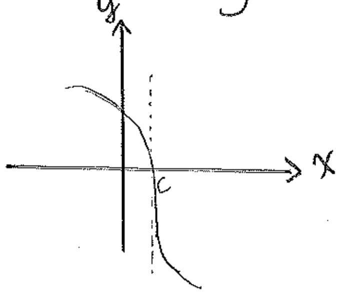
$f(c^+) \to \infty$

$f'(c^-) \to -\infty$

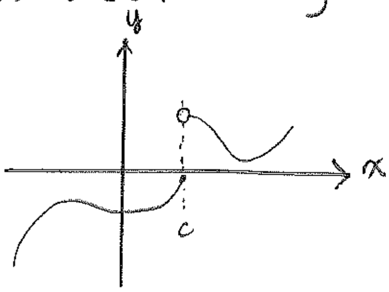
任3個, 1個(3%)

無文字說明只給2%

(c) vertical tangent



(d) discontinuouty



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5.(a) $\lim_{\theta\to\frac{\pi}{6}} \frac{\sin\theta-\frac{1}{2}}{\theta-\pi/6} = \lim_{\theta\to\pi/6} \frac{\sin\theta-\sin(\pi/6)}{\theta-\pi/6} = (\sin\theta)'\Big|_{\pi/6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ # **(8)**

(16%)

(b) $\frac{dy}{dt} = \frac{1}{2}(3t+\sqrt{2+\sqrt{1-t}})^{-\frac{1}{2}} \cdot \left[3+\frac{1}{2}(2+\sqrt{1-t})^{-\frac{1}{2}}\right) \cdot \left(\frac{1}{2}(1-t)^{-\frac{1}{2}}\right)\cdot(-1)\Big]$

$= \frac{1}{2}(3t+\sqrt{2+\sqrt{1-t}})^{-\frac{1}{2}}\left[3-\frac{1}{4}(2+\sqrt{1-t})^{-\frac{1}{2}})(1-t)^{-\frac{1}{2}}\right]$ **(8)**

6. $(f\circ g)' = \frac{df(u)}{du}\cdot\frac{du}{dx} = 2\left(\frac{2u-1}{2u+1}\right)\cdot\frac{(2u+1)\cdot 2-(2u-1)\cdot 2}{(2u+1)^2}\cdot\frac{du}{dx}$

(10%) $= \frac{8(2u-1)}{(2u+1)^3}\, g'(x)$

$g'(x) = \frac{-2}{x^3}$; $g'(-1)=2$ $u=g(-1)=0$.

$\therefore [f(g(x))]'\Big|_{x=-1} = \frac{8(0-1)}{(0+1)^3}\cdot 2 = -16$

7. (a) **(7)**

$\frac{d[y\cdot\sin(1/y)]}{dx} = \frac{d(1-xy)}{dx}$

(14%)

$\Rightarrow y\cdot\cos(1/y)\cdot\frac{-1}{y^2}\cdot\frac{dy}{dx} + \sin(1/y)\frac{dy}{dx} = -x\cdot\frac{dy}{dx} - y$

$\Rightarrow \left[\sin(1/y) - \frac{1}{y}\cos(1/y) + x\right]\frac{dy}{dx} = -y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{\sin(1/y)-\frac{1}{y}\cos(1/y)+x}$ #

(b) **(7)** $\frac{d[(x^2+y^2)^2]}{dx} = \frac{d(x-y)^2}{dx}$

$\Rightarrow \cancel{2}(x^2+y^2)(2x+2y\cdot\frac{dy}{dx}) = \cancel{2}(x-y)(1-\frac{dy}{dx})$

$\Rightarrow 2x(x^2+y^2)+2y(x^2+y^2)\frac{dy}{dx} = (x-y)-(x-y)\frac{dy}{dx}$

$\Rightarrow [2y(x^2+y^2)+(x-y)]\frac{dy}{dx} = -2x(x^2+y^2)+(x-y)$,

$\Rightarrow \frac{dy}{dx} = \frac{-2x(x^2+y^2)+(x-y)}{2y(x^2+y^2)+(x-y)}$. #

8. (a) $f'(x) = k(1+x)^{(k-1)}$; $f(0)=1$, $f'(0)=k$

(16%) $\therefore L(x) = 1+kx$ **(8)**

(b) $\sqrt[3]{1.009} = (1+0.009)^{\frac{1}{3}} \approx 1+\frac{1}{3}\times 0.009 = 1.003$ # **(8)**