THOMAS' CALCULUS (12/E)

15.2 Double Integrals over General Regions

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Double Integrals over Bounded, Nonrectangular Regions

1.1 *Theorem: Fubini's Theorem (Stronger Form)*

Let $f(x,y)$ be continuous on a region R.

(a) If R is defined by $a \le x \le b, g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a,b]$, then

$$\iint_R f(x,y)\,dA = \underline{\qquad\qquad\qquad\qquad}.$$

(b) If R is defined by $c \le y \le d, h_1(y) \le x \le h_2(y)$, with h_1 and h_2 continuous on $[c,d]$, then

$$\iint_R f(x,y)\,dA = \underline{\qquad\qquad\qquad\qquad}.$$

✎ Ex. 1 ..... (example1, p843)

Find the volume of the prism whose base is the triangle in the xy-plane bounded by hte x-axis and the line $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x,y) = 3 - x - y$.

sol:

✎ Ex. 2 ..... (example2, p844)

Calculate $\iint_R \frac{\sin x}{x}\, dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$, and the line $x = 1$.

sol:

2 Finding Limits of Integration

2.1 *Using Vertical Cross-sections*

When faced with evaluating $\iint_R f(x, y)\, dA$, integrating first with respect to y and then with respect to x, do the following:

(a) Sketch the _____ and label the _____.

(b) Find the _____ of integration. (c) Find the _____ of integration.

2.2 *Using Horizontal Cross-sections*

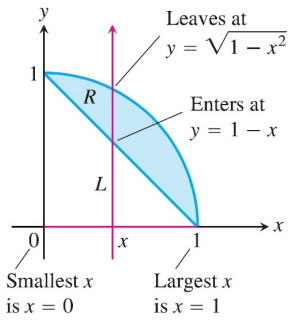

FIGURE 15.14 Finding the limits of integration when integrating first with respect to y and then with respect to x.

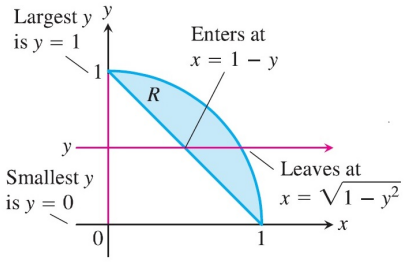

FIGURE 15.15 Finding the limits of integration when integrating first with respect to x and then with respect to y.

2.3 *Properties of Double Integrals*

If $f(x,y)$ and $g(x.y)$ are continuous on the bouneded region R, then

(a) Constant multiple: $\iint_R kf(x,y)\ dA =$ _____ .

(b) Sum and difference: $\iint_R [f(x,y) \pm g(x,y)]\ dA =$ _____ .

(c) Additivity: $\iint_R f(x,y)\ dA =$ _____ .

(d) Domination: If $f(x,y) \geq g(x,y)$, then _____ .

Ex. 3 ..... (example3, p846)

$\int_0^2 \int_{x^2}^{2x} (4x+2)\ dydx$

sol:

Ex. 4 ..... (example4, p847)

Find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$, the line $y = 4x - 2$, and the x-axis.

sol:

實習課練習 (EXERCISE 15.2)

19. $\int_0^{\pi} \int_0^{x} x \sin y \; dydx.$

21. $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} \; dxdy.$

25. Integrate f over the given region: $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$ and $x = 2$.

36. $\int_0^{1} \int_{1-x}^{1-x^2} \; dydx.$

43. $\int_1^{e} \int_0^{\ln x} xy \; dydx.$

44. $\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2 \; dydx.$

47. $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} \; dydx.$

49. $\int_0^{1} \int_y^{1} x^2 e^{xy} \; dxdy.$

60. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.