THOMAS' CALCULUS (12/E)

# 7.3 Exponential Function

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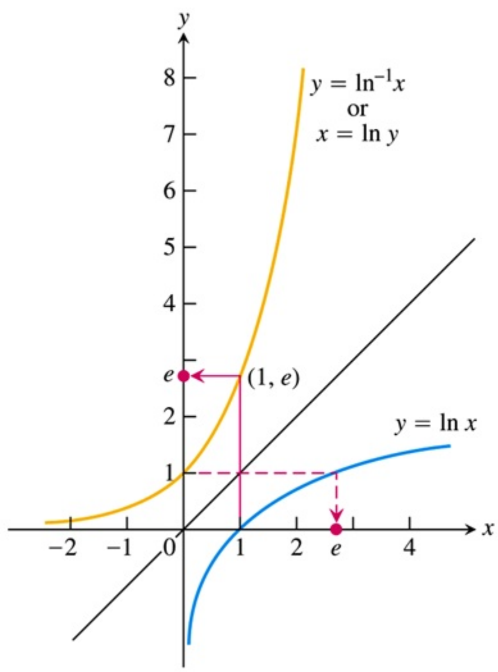
## 1 The Inverse of $\ln x$ and $e^x$

1.1 The function $\ln x$, being an increasing function of $x$ with domain $(0, \infty)$ and range $(-\infty, \infty)$, has an inverse $\ln^{-1} x$ with domain $(-\infty, \infty)$ and range $(0, \infty)$. (圖示如下)




1.2 $\ln(e) =$______, $e =$ ____________ $=$ ___________.

1.3 $e \doteq$ ________________________.

1.4 *Definitions: The Natural Exponential Function*

> For every real number $x$, $e^x =$ __________ $=$ __________.

1.5 $\ln e^r =$ __________________ $\Rightarrow$ $e^r =$ __________________.

1.6 Inverse Equation for $e^x$ and $\ln x$

$$e^{lnx} = \_\_\_\_\_\_, \quad \forall x > 0$$

$$\ln(e^x) = \_\_\_\_\_\_, \quad \forall x$$

**Ex. 1** ................................................ (`example1, p378`)

Solve the equation $e^{2x-6} = 4$ for $x$.

*sol:*

## 2 The Derivative and Integral of $e^x$

2.1 Let $f(x) = \ln x$ and $y = e^x =$ __________________. Then $\frac{dy}{dx} =$ _______

________________________________________________________

2.2 If $u$ is any differentiable function of $x$, then $\frac{d}{dx} e^u =$ __________

2.3 $\int e^u \, du =$ __________

**Ex. 2** ................................................ (`example2, p379`)

(a) $\frac{d}{dx}(5e^x) =$

(b) $\frac{d}{dx} e^{-x} =$

(c) $\dfrac{d}{dx}\, e^{\sin x} =$

(d) $\dfrac{d}{dx}\, (e^{\sqrt{3x+1}}) =$

✎ Ex. 3 ............................................. (example3, p379)

(a) $\displaystyle\int_0^{\ln 2} e^{3x}\, dx =$

(b) $\displaystyle\int_0^{\pi/2} e^{\sin x} \cos x\, dx =$

# 3 Laws of Exponents

3.1 *Definitions: General Exponential Functions*

> For any numbers $a > 0$ and $x$, the exponential function with base $a$ is _____________.

3.2 *Theorem 3: Laws of Exponents for $e^x$*

> For all numbers $x, x_1$ and $x_2$, the natural exponential $e^x$ obeys the following laws:
>
> (a) $e^{x_1} \cdot e^{x_2} =$__________
>
> (b) $e^{-1} =$_________
>
> (c) $e^{x_1}/e^{x_2} =$__________
>
> (d) $((e^{x_1})^{x_2}) =$_________________

Proof of Law (a):

✎ Ex. 4 ................................................ (example4, p382)

Differentiate $f(x) = x^x,\ x > 0$.

*sol:*

# 4 The Number $e$ Expressed as a Limit

4.1 *Theorem 4: The Number e as a Limit*

The number $e$ can be calculated as the limit: $e =$ ________________.

**Proof:**

4.2 *Power Rule (General Form)*

If $u$ is a positive differentiable function of $x$ and $n$ is any real number, then $u^n$ is a differentiable function of $x$ and $\dfrac{d}{dx}u^n =$ ____________.

4.3 Examples:

(a) $\dfrac{d}{dx}x^{\sqrt{2}} =$

(b) $(2 + \sin 3x)^\pi =$

# 5 The Derivative of $a^u$

5.1 If $a > 0$, then $\dfrac{d}{dx}a^x =$__________.

**Proof:**

5.2 If $a > 0$ and $u$ is a differentiable function of $x$, then $a^u$ is a differentiable function of $x$ and $\frac{d}{dx}a^u = $ ____________.

5.3 If $a \neq 1$, $\int a^u \, du = $ ____________.

**Proof:**

✎ Ex. **5** ............................................. (example5, p383)

(a) $\frac{d}{dx}\, 3^x =$

(b) $\frac{d}{dx}\, 3^{-x} =$

(c) $\frac{d}{dx}\, 3^{\sin x} =$

(d) $\int 2^x \, dx =$

(e) $\int 2^{\sin x} \cos x \, dx =$

# 6 Logarithms with Base $a$

6.1 *Definitions:* $\log_a x$

For any positive number $a \neq 1$, ________ is the inverse function of $a^x$.

6.2 Inverse Equations for $a^x$ and $log_a x$

$a^{\log_a x} =$______, $x > 0$

$\log_a(a^x) =$______, $\forall x$

6.3 $\log_a x =$ ________

**Proof:**

6.4 Derivatives and Integrals Involving $\log_a x$:

$$\frac{d}{dx}(\log_a u) = \underline{\hspace{3cm}}$$

Ex. **6** .................................................. (example6, p385)

(a) $\frac{d}{dx} \log_{10}(3x + 1) =$

(b) $\int \frac{\log_2 x}{x}\, dx =$

# 實習課練習 (EXERCISE 7.3)

☐ Solve for $t$.

**2.** (a) $e^{-0.01t} = 1000$, (b) $e^{kt} = \frac{1}{10}$, (c) $e^{(\ln 2)x} = 1/2$.

**4.** $e^{x^2}e^{2x+1} = e^t$

☐ Find the derivative of $y$ with respect to $x$, $t$ or $\theta$, as appropriate.

**9.** $y = xe^x - e^x$

**14.** $y = \ln(3\theta e^{-\theta})$

**20.** $y = \ln(\dfrac{\sqrt{\theta}}{1+\sqrt{\theta}})$

**24.** $y \displaystyle\int_{e^4\sqrt{x}}^{e^{2x}} \ln t \; dt$

**25.** $\ln y = e^y \sin x$

**28.** $\tan y = e^x + \ln x$

☐ Evaluate the integrals.

**33.** $\displaystyle\int 8e^{x+1} \; dx$

**41.** $\displaystyle\int \frac{e^{1/x}}{x^2} \; dx$

**43.** $\displaystyle\int_0^{\pi/4} (1 + e^{\tan\theta}) \sec^2\theta \; d\theta$

**49.** $\displaystyle\int \frac{e^r}{1+e^r} \; dr$

**50.** $\displaystyle\int \frac{dx}{1+e^x}$

☐ Find the derivative of $y$ with respect to the given independent variable.

**57.** $y = 5^{\sqrt{s}}$

**72.** $y = \log_3 r \cdot \log_9 r$

**81.** $y = \log_2(8t^{\ln 2})$

☐ Evaluate the integrals.

**85.** $\displaystyle\int_0^1 2^{-\theta}\ d\theta$

**91.** $\displaystyle\int_2^4 x^{2x}(1+\ln x)\ dx$

**96.** $\displaystyle\int_1^e x^{(\ln 2 - 1)}\ dx$

**99.** $\displaystyle\int_1^4 \frac{\ln 2 \log_2 x}{x}\ dx$

**106.** $\displaystyle\int \frac{dx}{x(\log_8 x)^2}$

**107.** $\displaystyle\int_1^{\ln x} \frac{1}{t}\ dt, \quad x > 1$

☐ Find the derivative of $y$ with respect to the given independent variable.

**111.** $y = x + 1^x$

**116.** $y = x^{\sin x}$

**118.** $y = (\ln x)^{\ln x}$