Quiz (2) - page 1

1. (a) $\lim_{x\to\infty}\frac{1-x^2}{x^2+1}=\lim_{x\to\infty}\frac{\frac{1}{x^2}-1}{1+\frac{1}{x^2}}=\frac{-1}{1}=-1$

$\lim_{x\to-\infty}\frac{1-x^2}{x^2+1}=\lim_{x\to-\infty}\frac{\frac{1}{x^2}-1}{1+\frac{1}{x^2}}=\frac{-1}{1}=-1$ $\boxed{y=-1\text{ 為 }f(x)\text{ 之水平漸近線}}$

∵分母 x^2+1 恒正 ⇒ 垂直漸近線不存在

(10%)

(b) $f(x)=\frac{x^2-1}{2x+4}=\frac{1}{2}\cdot\frac{x^2-1}{x+2}=\frac{1}{2}\left[(x-2)+\frac{3}{x+2}\right]$

當 $x\to\infty$，$f(x)\to\frac{1}{2}(x-2)$ ⇒ $\boxed{y=\frac{1}{2}x-1\text{ 為 }f(x)\text{ 之漸近線}}$

當 $x\to-\infty$，$f(x)\to\frac{1}{2}(x-2)$

(15%)

∵ $x=-2$ 時分母 $2x+4=0$

⇒ $\lim_{x\to-2^-}\frac{x^2-1}{2x+4}=-\infty$；$\lim_{x\to 2^+}\frac{x^2-1}{2x+4}=\infty$ ⇒ $\boxed{x=-2\text{ 為 }f(x)\text{ 之水平漸近線}}$

2. (a) $f'(x_0)=\lim_{h\to0}\frac{f(x_0+h)-f(x_0)}{h}$. (10%)

(b) to prove: f 在 x_0 可微 → f 在 x_0 連續

(15%)

$$\lim_{h\to0}f(x_0+h)=\lim_{h\to0}\left(f(x_0+h)+f(x_0)-f(x_0)\right)$$
$$=\lim_{h\to0}\left[f(x_0)+\frac{f(x_0+h)-f(x_0)}{h}h\right]$$
$$=\lim_{h\to0}f(x_0)+\lim_{h\to0}\left(\frac{f(x_0+h)-f(x_0)}{h}\right)\lim_{h\to0}h.$$
$$=f(x_0)+f'(x_0)\cdot0=f(x_0)$$ ※ 得証 $f(x)$ 在 x_0 連續

3. (a) $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}$

$$=\lim_{h\to0}\frac{\left(\sqrt{(2x+1)+2h}-\sqrt{2x+1}\right)\left(\sqrt{(2x+1)+2h}+\sqrt{2x+1}\right)}{h\cdot\left(\sqrt{(2x+1)+2h}+\sqrt{2x+1}\right)}$$

(15%)

$$=\lim_{h\to0}\frac{(2x+1+2h)-(2x+1)}{h\cdot\left(\sqrt{(2x+1)+2h}+\sqrt{2x+1}\right)}=\lim_{h\to0}\frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+1+2h}+\sqrt{2x+1}\right)}$$

※不用定義只拿(1%)

$$=\frac{2}{2\sqrt{2x+1}}=\frac{1}{\sqrt{2x+1}}\ \#$$

(b) $g'(x)=\lim_{h\to0}\frac{g(x+h)-g(x)}{h}=\lim_{h\to0}\frac{\left(\frac{x+h}{x+h-1}\right)-\left(\frac{x}{x-1}\right)}{h}=\lim_{h\to0}\frac{(x+h)(x-1)-x(x+h-1)}{(x+h-1)(x-1)\cdot h}$

$$=\lim_{h\to0}\frac{x^2+hx-x-h-x^2-hx+x}{(x+h-1)(x-1)h}=\lim_{h\to0}\frac{-\cancel{h}}{(x+h-1)(x-1)\cancel{h}}=\lim_{h\to0}\frac{-1}{(x+h-1)(x-1)}=\frac{-1}{(x-1)^2}\ \#$$

(10%)

Quiz(2) – page 2

4. $y = \dfrac{(\sin x+1)(\cos x+2)}{(\tan x-1)(\sec x-2)}$.

註: $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{g(x)\cdot f'(x) - f(x)g'(x)}{[g(x)]^2}$.

$$\frac{dy}{dx} = \frac{(\tan x-1)(\sec x-2)\cdot\left[\cos x(\cos x+2)-\sin x(\sin x+1)\right] - (\sin x+1)(\cos x+2)\left[\sec^2 x\cdot(\sec x-2) + \sec x\tan x(\tan x-1)\right]}{(\tan x-1)^2(\sec x-2)^2}$$

(25%) 寫到此即有完整分數

5. let $f(x) = x^{\frac{2}{9}}$

$$\lim_{x\to -1}\frac{x^{\frac{2}{9}}-1}{x+1} = \lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)} = \underline{f'(-1)}.$$

$\because f'(x) = \frac{2}{9}x^{\frac{-7}{9}}$; $f(-1) = \frac{2}{9}\cdot(-1) = \frac{-2}{9}$ #.

(25%) [使用羅必達法則不給分]