

Microarray Data Analysis

Clustering and Visualization

國立台灣大學資訊所

Course: 生物資訊與計算分子生物學

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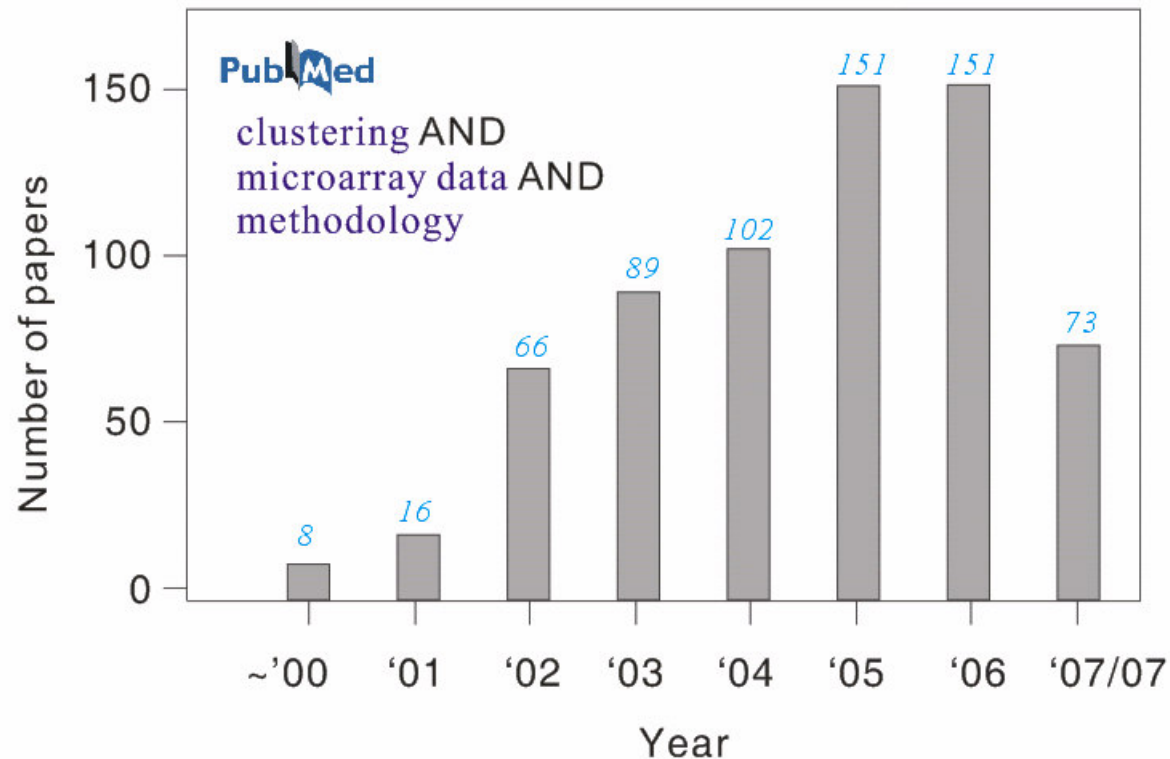
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中央研究院 統計科學研究所
Institute of Statistical Science, Academia Sinica

Clustering of Microarray Gene Expression Data

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A continuing and active topic of research and application!

Outlines

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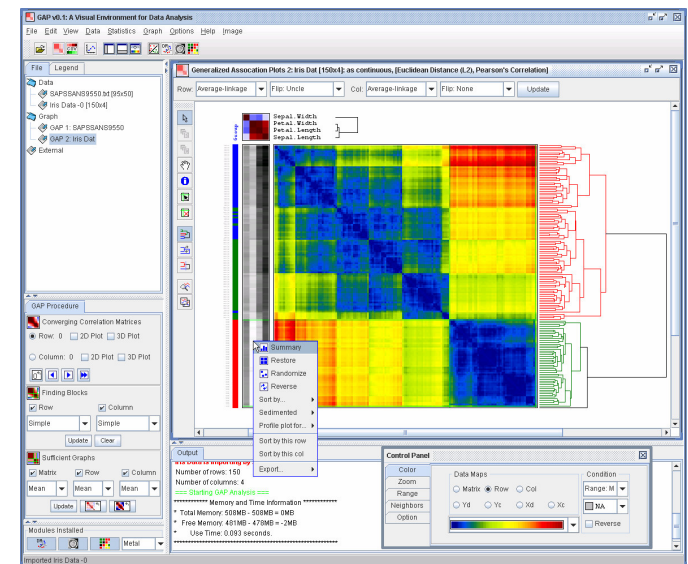
- ◆ **Overview of Microarray Experiment**
- ◆ **Clustering Analysis and Visualization**
- ◆ **Distance and Similarity Measure**
- ◆ **K-Means Clustering**

- ◆ **Visualizing Clustering Results: Dimension Reduction Techniques**
 - Principal Component Analysis (PCA)
 - Multidimensional Scaling (MDS)

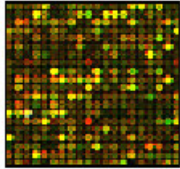
- ◆ **Clustering Analysis and Visualization**
 - Self-Organizing Maps (SOM)
 - Heat Map
 - Hierarchical Clustering
 - Tree Flip

- ◆ **Cluster Validation**
- ◆ **Software**

GAP



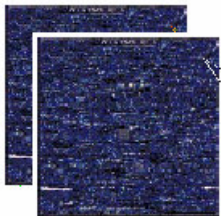
Overview of Microarray Experiment



cDNA Microarray Data

	A	B	C	D
1	UNIQUID	Gene Name Description	Array 1	Array 2
2	588029	588029:Hs.79:ACY1	0.645	0.375
3	190929	190929:Hs.247565:RHO	0.615	0.210
4	246550	246550:Hs.293548	0.585	0.665
5	32553	32553:Hs.101248	0.825	0.230
6	446172	446172:	0.570	0.495
7	417978	417978:Hs.268874	0.495	1.835
...				
12000	366879	366879:Hs.169341	1.835	0.300

$\log_2(\text{Cy5}/\text{Cy3})$

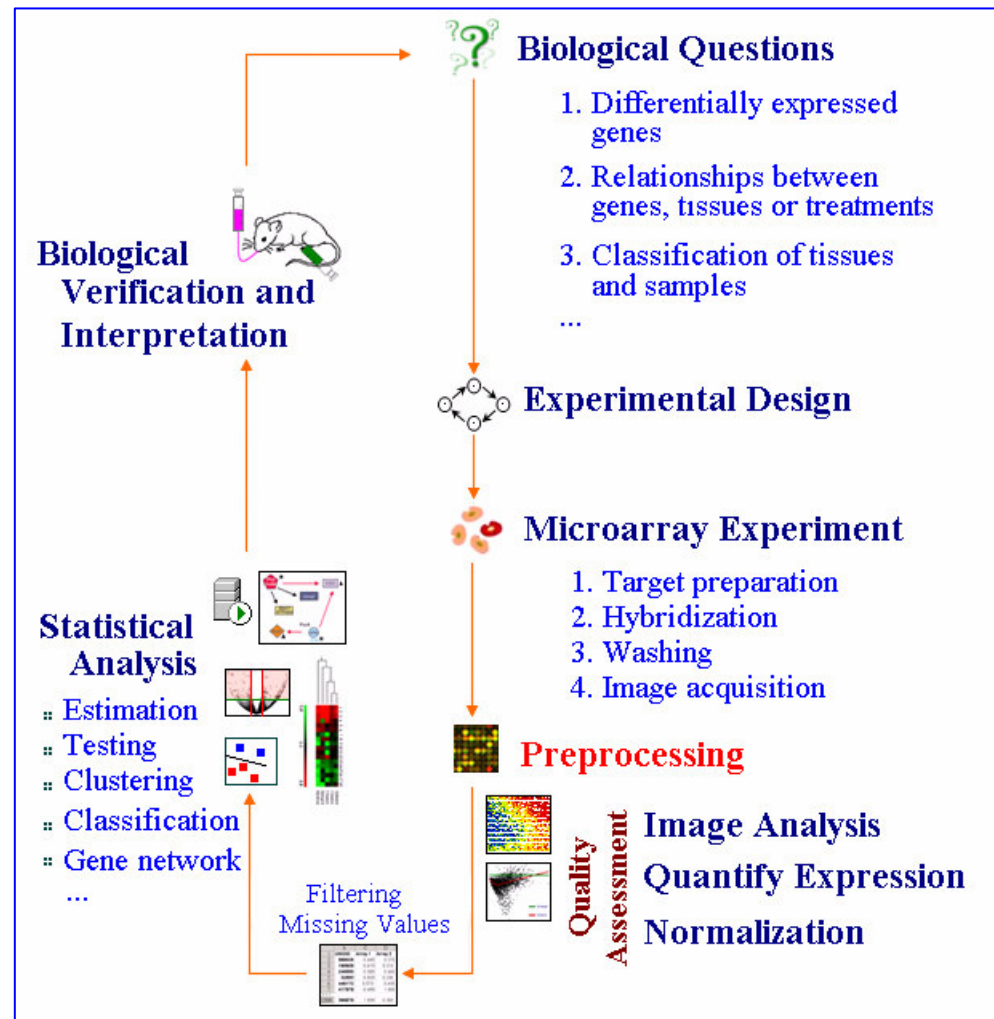


Oligonucleotide Array Data

	A	B	C	D
1	Probeset	Gene Name	Array 1	Array 2
2	103941_at	alpha-spectin 1, erythroid	33.7625	29.2333
3	104432_at	aplysia ras-related homolog N (Rho)	127.736	99.6895
4	104137_at	ATP-binding cassette, sub-family 4	109.522	65.2727
5	98458_at	baculoviral IAP repeat-containing 5	128.96	123.371
6	93243_at	bone morphogenetic protein 7	174.85	174.019
7	95061_at	breast carcinoma amplified sequer	34.8	43.6696
...				
12600	102632_at	calmodulin binding protein 1	69.688	54.7391

Expression index

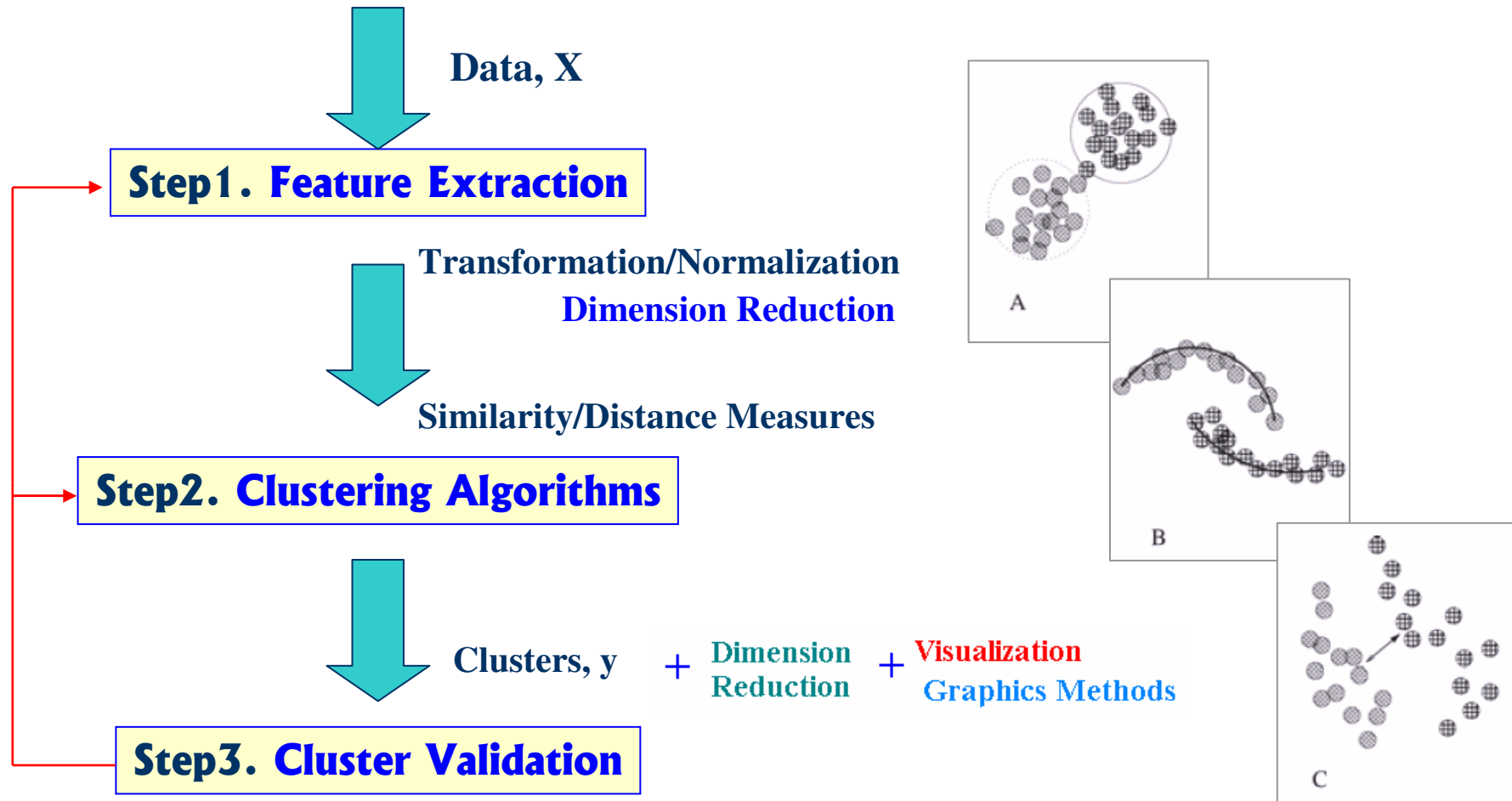
Microarray Life Cycle



Cluster Analysis (Unsupervised Learning)

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Group a given collection of **unlabeled** patterns into **meaningful** clusters.



Daxin Jiang, Chun Tang and Aidong Zhang, (2004), **Cluster analysis for gene expression data: a survey**, IEEE Transactions on Knowledge and Data Engineering 16(11), 1370- 1386.

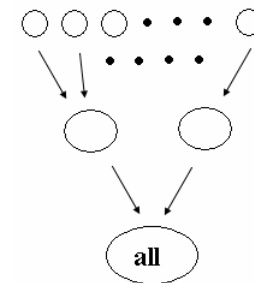
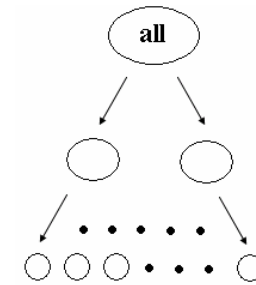
Clustering Analysis

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Hierarchical clustering

The result is a tree that depicts the relationships between the objects.

- **Divisive clustering:**
begin at step 1 with all the data in one cluster.
- **Agglomerative clustering:**
all the objects start apart., there are n clusters at step 0.



Non-Hierarchical clustering

- k-means, The EM algorithm, K Nearest Neighbor,...

Two important properties of a clustering definition:

1. Most of data has been organized into **non-overlapping clusters**.
2. Each cluster has a within variance and one between variance for each of the other clusters. A good cluster should have a **small within variance** and **large between variance**.

Data/Information Visualization

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What is Visualization?

- To visualize = to make visible, to transform into pictures.
- Making things/processes visible that are not directly accessible by the human eye.
- Transformation of an abstraction to a picture.
- Computer aided extraction and display of information from data.

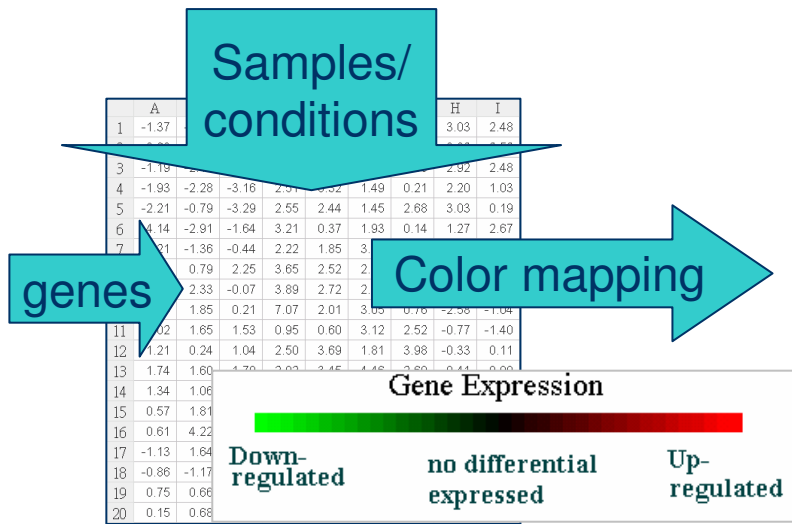
Data/Information Visualization

- Exploiting the human visual system to extract information from data.
- Provides an overview of complex data sets.
- Identifies structure, patterns, trends, anomalies, and relationships in data.
- Assists in identifying the areas of interest.

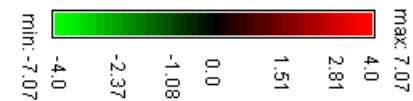
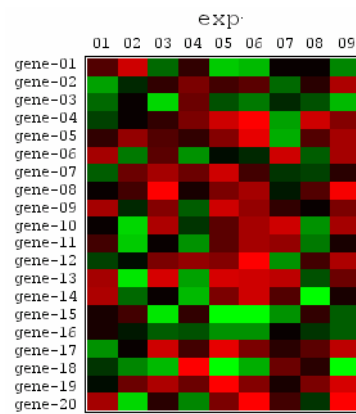
Visualization = Graphing for Data + Fitting + Graphing for Model

Tegarden, D. P. (1999). Business Information Visualization. Communications of AIS 1, 1-38.

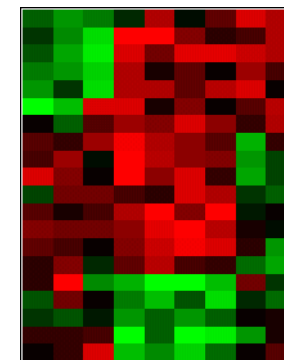
Visualizing Clustering Results: Heat Map



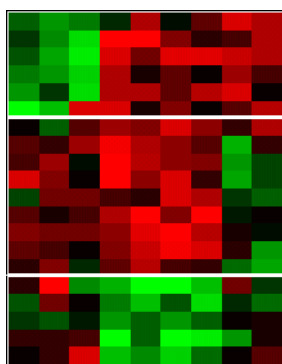
Without ordering



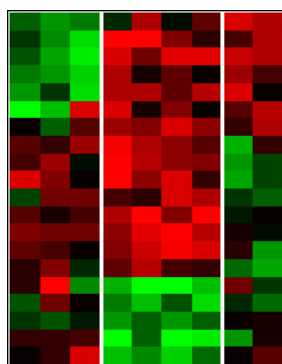
Ordering/
Seriation/
Clustering



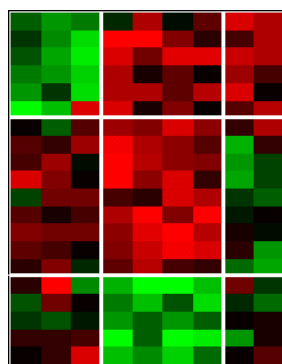
Gene-based
clustering



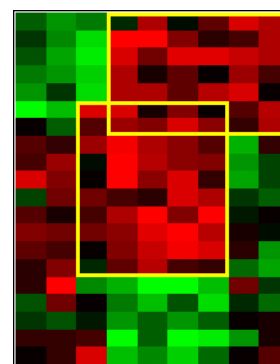
Sample-based
clustering



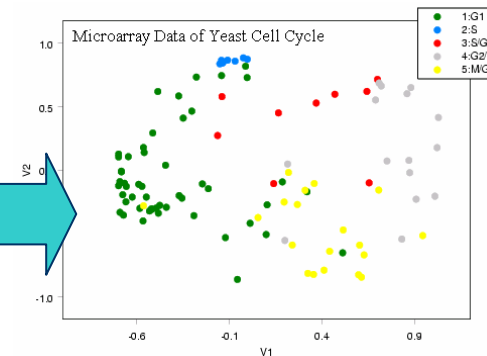
Twoway-based
clustering



Subspace
clustering



Dimension Reduction



e.g., K-means, SOM, Hierarchical Clustering,
Model-based clustering,...

e.g., Bi-clustering

Clustering Analysis in Microarray Experiments

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Goals

- Find natural classes in the data
- Identify new classes/gene correlations
- Refine existing taxonomies
- Support biological analysis/discovery

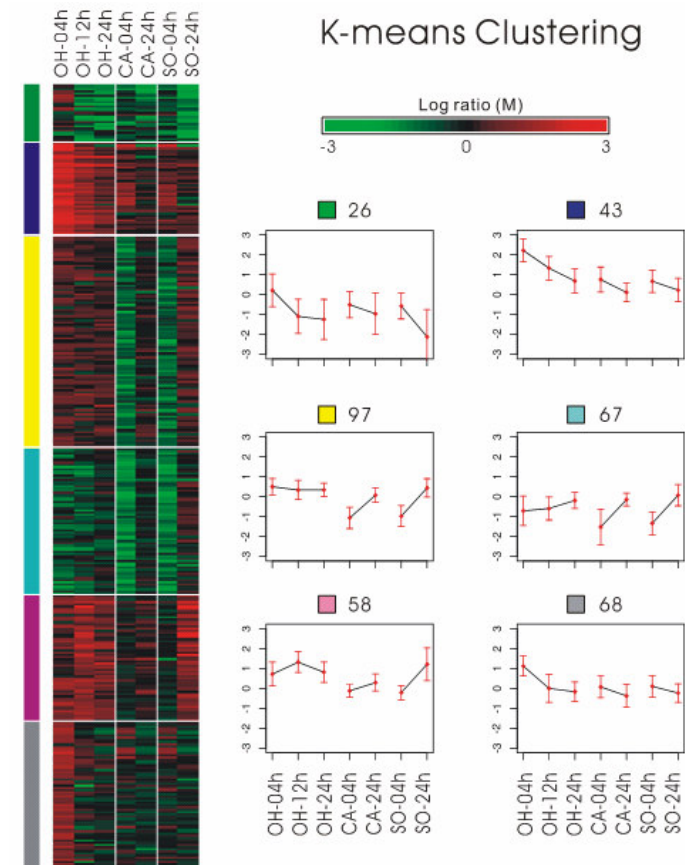
- cluster genes based on samples profiles
- cluster samples based on genes profiles

Hypothesis:

- genes with similar function have similar expression profiles.
- Clustering results in groups of co-expressed genes, groups of samples with a common phenotype, or blocks of genes and samples involved in specific **biological processes**.

Characteristic of Microarray Data:

- High-throughput, Noise, Outliers



Distance and Similarity Measure

Cov	x1	x2	x3	x4	x p
x1	1.00	0.48	0.10	-0.10	-0.28
x2	0.48	1.00	0.41	0.22	-0.23
x3	0.10	0.41	1.00	0.36	-0.05
x4	-0.10	0.22	0.36	1.00	0.10
x p	-0.28	-0.23	-0.05	0.10	1.00

Proximity Matrix

Pearson Correlation Coefficient

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Data Matrix

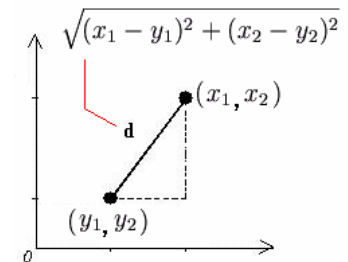
Data	x1	x2	x3	x4	...	x p
subject01	-0.48	-0.42	0.87	0.92	...	-0.18
subject02	-0.39	-0.58	1.08	1.21	...	-0.33
subject03	0.87	0.25	-0.17	0.18	...	-0.44
subject04	1.57	1.03	1.22	0.31	...	-0.49
subject05	-1.15	-0.86	1.21	1.62	...	0.16
subject06	0.04	-0.12	0.31	0.16	...	-0.06
subject07	2.95	0.45	-0.40	-0.66	...	-0.38
subject08	-1.22	-0.74	1.34	1.50	...	0.29
subject09	-0.73	-1.06	-0.79	-0.02	...	0.44
subject10	-0.58	-0.40	0.13	0.58	...	0.02
subject11	-0.50	-0.42	0.66	1.05	...	0.06
subject12	-0.86	-0.29	0.42	0.46	...	0.10
subject13	-0.16	0.29	0.17	-0.28	...	-0.55
subject14	-0.36	-0.03	-0.03	-0.08	...	-0.25
subject15	-0.72	-0.85	0.54	1.04	...	0.24
subject16	-0.78	-0.52	0.26	0.20	...	0.48
subject17	0.60	-0.55	0.41	0.45	...	-0.66
⋮						
subject n	-2.29	-0.64	0.77	1.60	...	0.55
mean	0.07	-0.04	0.44	0.31	...	-0.21

$$x = (x_1, x_2, \dots, x_n)$$

$$y = (y_1, y_2, \dots, y_n)$$

Euclidean Distance

$$d_{xy} = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$



- The standard transformation from a similarity matrix C to a distance matrix D is given by $d_{rs} = (c_{rr} - 2c_{rs} + c_{ss})^{1/2}$.
- (Eisen *et al.* 1998) $d_{rs} = 1 - c_{rs}$
- Other transformations (Chatfield and Collins 1980, Section 10.2)

More Similarity Measures

Dissimilarity/Similarity Measure for Quantitative Data

Kendall's tau

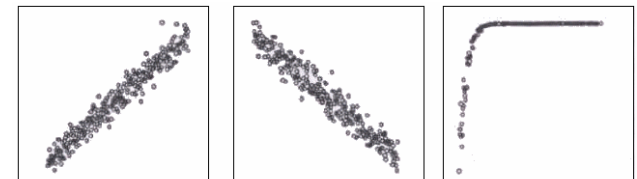
Two pairs of observation (x_i, y_i) and (x_j, y_j)

- C: concordant pair: $(x_j - x_i)(y_j - y_i) > 0$
- D: discordant pair: $(x_j - x_i)(y_j - y_i) < 0$
 - tie:
- E_y : extra y pair in x 's: $(x_j - x_i) = 0$
- E_x : extra x pair in y 's: $(y_j - y_i) = 0$

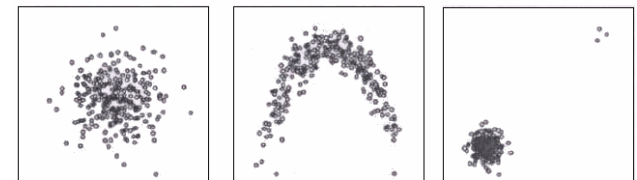
$$\tau = \frac{C - D}{\sqrt{C + D - E_y} \sqrt{C + D - E_x}}$$

- Pearson's rho measures the strength of a linear relationship [(a), (b)].
- Spearman's rho and Kendall's tau measure any monotonic relationship between two variables [(a), (b), (c)].
- If the relationship between the two variables is non-monotonic, all three correlation coefficients fail to detect the existence of a relationship [(e)].
- Both Spearman's rho and Kendall's tau are rank-based non-parametric measures of association between variable X and Y.
- The rank-based correlation coefficients are more robust against outliers.

Similarity	Formula
Pearson correlation	$s(i, j) = \frac{\text{cov}(x_i, x_j)}{\sqrt{\text{var}(x_i) \text{var}(x_j)}}$
Spearman correlation (r_i is ranked x_i)	$s(i, j) = \frac{\text{cov}(r_i, r_j)}{\sqrt{\text{var}(r_i) \text{var}(r_j)}}$
Kendall's Tau	$s(i, j) = \frac{1}{\binom{p}{2}} \sum_{k \neq k'} \text{sign} [(x_{ik} - x_{ik'})(x_{jk} - x_{jk'})]$



(a) positive linear correlation (b) negative linear correlation (c) nonlinear relationships



(d) no relationship (e) nonlinear relationships (f) no relationship with outliers

Data	Pearson's rho	Spearman's rho	Kendall's tau
(a)	0.98	0.98	0.87
(b)	-0.98	-0.98	-0.87
(c)	0.50	0.99	0.98
(d)	-0.02	-0.03	-0.02
(e)	-0.06	-0.02	-0.02
(f)	0.68	0.00	0.00

Algorithm they use different logic for computing the correlation coefficient, they seldom lead to markedly different conclusions (Siegel and Castellan, 1988).

K-Means Clustering

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- K-means is a **partition methods** for clustering.
- Data are classified into **k groups** as specified by the user.
- Two different clusters cannot have any objects in common, and the k groups together constitute the full data set.

Optimization problem:

Minimize the sum of squared within-cluster distances

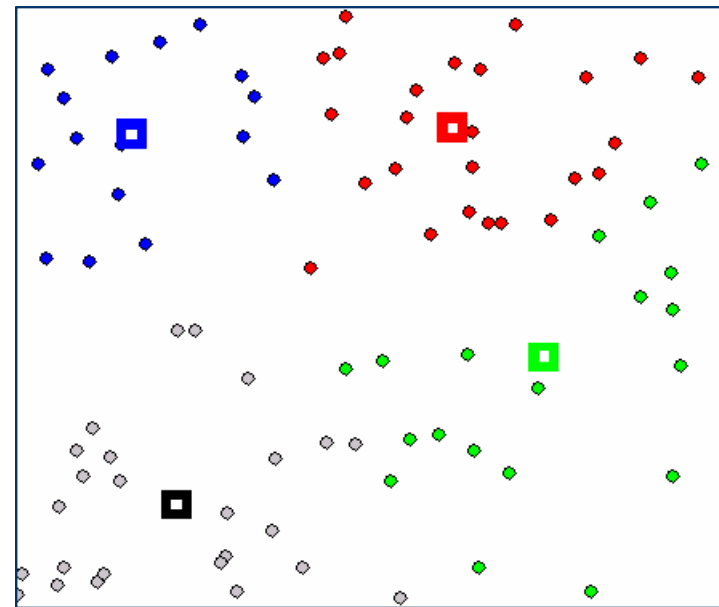
The K-Means Algorithm

1. The data points are randomly assigned to one of the K clusters.
2. The position of the K centroids are determined (initial group centroids).
3. For each data point:
 - Calculate the distance from the data point to each cluster.
 - Assign data point to the cluster that has the closest centroid.
4. Repeat the above step until the centroids no longer move.

The choice of initial partition can greatly affect the final clusters that result.

$$W(C) = \frac{1}{2} \sum_{k=1}^K \sum_{C(i)=C(j)=k} d_E(x_i, x_j)^2$$

Converged



Visualizing Clustering Results:

Dimension Reduction Techniques

- ◆ Principal Component Analysis (PCA)
- ◆ Multidimensional Scaling (MDS)

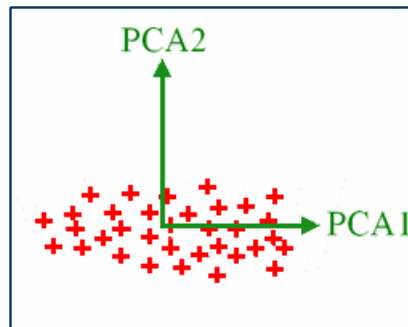
Dimension reduction visualization is often adopted for presenting grouping structure for methods such as K-means.

Principal Component Analysis (PCA)

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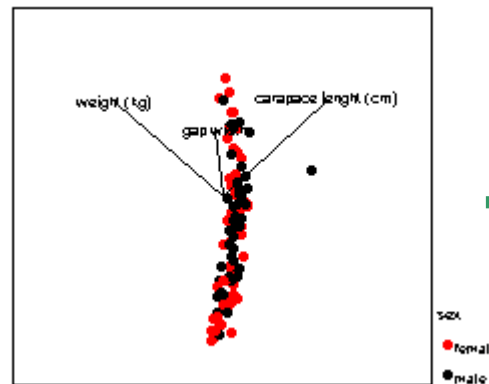
(Pearson 1901; Hotelling 1933; Jolliffe 2002)

PCA is a method that reduces data dimensionality by finding the new variables (major axes, principal components).



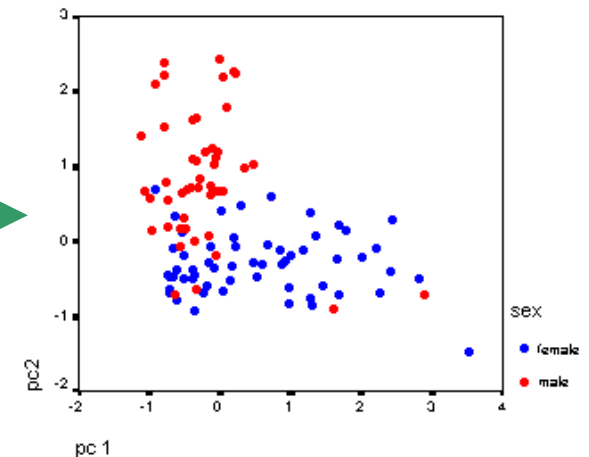
$$PCA_1 = a_1 X + b_1 Y$$

$$PCA_2 = a_2 X + b_2 Y$$



$$PCA_1 = a_1 X + b_1 Y + c_1 Z$$

$$PCA_2 = a_2 X + b_2 Y + c_2 Z$$



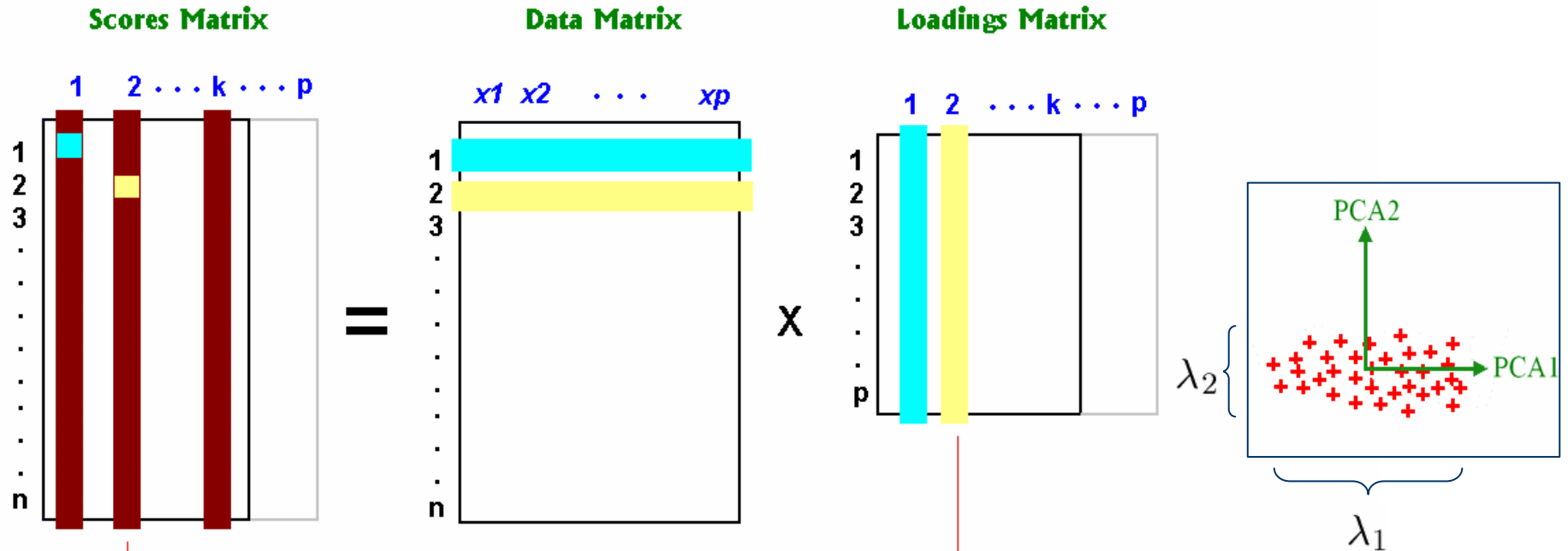
Amongst all possible projections, PCA finds the projections so that the maximum amount of information, measured in terms of variability, is retained in the smallest number of dimensions.

$$PCA_1 = a_{11} X_1 + a_{12} X_2 + \dots + a_{1p} X_p$$

$$PCA_2 = a_{21} X_1 + a_{22} X_2 + \dots + a_{2p} X_p$$

PCA: Loadings and Scores

$$\mathbf{Z} = \mathbf{X} \mathbf{W}$$



The i th principal component of \mathbf{X} is $\mathbf{X}\mathbf{w}_i$, where \mathbf{w}_i is the i th normalized eigenvector of $\Sigma_{\mathbf{x}}$ corresponding to the i th largest eigenvalue.

Eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$

$$\text{proportion} = \frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^p \lambda_i}$$

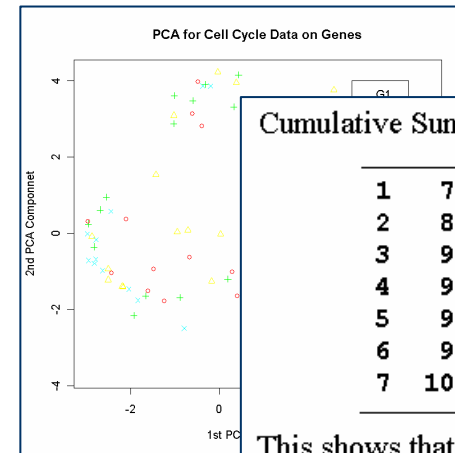
PCA (conti.)

Microarray Data Matrix

MA Table	exp01	exp02	exp03	exp04	exp05	exp...	exp P
gene001	-0.48	-0.42	0.87	0.92	0.67		-0.35
gene002	-0.39	-0.58	1.08	1.21	0.52		-0.58
gene003	0.87	0.25	-0.17	0.18	-0.13		-0.13
gene004	1.57	1.03	1.22	0.31	0.16		-1.02
gene005	-1.15	-0.86	1.21	1.62	1.12		-0.44
gene006	0.04	-0.12	0.31	0.16	0.17		0.08
gene007	2.95	0.45	-0.40	-0.66	-0.59		-0.76
gene008	-1.22	-0.74	1.34	1.50	0.63		-0.55
gene009	-0.73	-1.06	-0.79	-0.02	0.16		0.03
gene010	-0.58	-0.40	0.13	0.58	-0.09		-0.45
gene011	-0.50	-0.42	0.66	1.05	0.68		0.01
gene012	-0.86	-0.29	0.42	0.46	0.30		-0.63
gene013	-0.16	0.29	0.17	-0.28	-0.02		-0.04
gene014	-0.36	-0.03	-0.03	-0.08	-0.23		-0.21
gene015	-0.72	-0.85	0.54	1.04	0.84		-0.64
gene016	-0.78	-0.52	0.26	0.20	0.48		0.27
gene017	0.60	-0.55	0.41	0.45	0.18		-1.02
gene018	-0.20	-0.67	0.13	0.10	0.38		0.05
gene019	-2.29	-0.64	0.77	1.60	0.53		-0.38
gene020	-1.46	-0.76	1.08	1.50	0.74		-0.70
gene021	-0.57	0.42	1.03	1.35	0.64		-0.40
gene022	-0.11	0.13	0.41	0.60	0.23		0.19
gene...							
gene n	-1.79	0.94	2.13	1.75	0.23		-0.66

PCA on Conditions

MA Table	PCA-1	PCA-2	PCA-3
gene001	-0.18	-0.11	-0.03
gene002	0.51	-0.53	0.54
gene003	-0.35	-0.39	0.26
gene004	-0.18	-1.08	0.41
gene005	-0.62	-0.8	0.13
gene006	-0.09	-0.23	0.77
gene007	-0.38	-0.32	1.08
gene008	-0.88	-0.55	1.03
gene009	-1.26	0.45	0.41
gene010	0.12	-0.36	-0.16
gene011	-0.28	-0.44	2.13
gene012	-0.45	-0.23	0.82
gene013	-0.2	-0.43	0.44
gene014	0.03	-0.26	-0.68
gene015	-0.7	-0.76	0.5
gene016	-0.61	0.07	-0.04
gene017	-0.23	-0.71	0.01
gene018	0.1	0.1	0.11
gene019	-0.94	-0.97	0.24
gene020	-0.55	-0.53	0.86
gene021	-0.47	-0.87	-0.02
gene022	-0.34	-1.1	0.51
gene...	-0.49	-0.2	0.91
gene n	-0.15	-1.04	-0.01

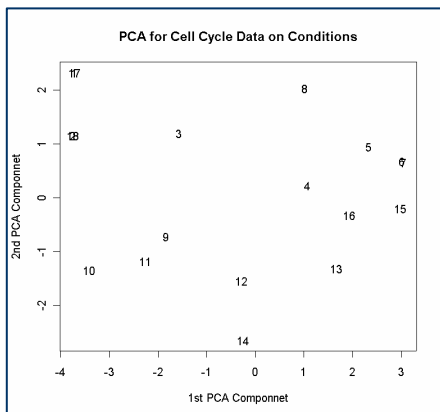


Cumulative Sum of the Variances:

1	78.3719
2	89.2140
3	93.4357
4	96.0831
5	98.3283
6	99.3203
7	100.0000

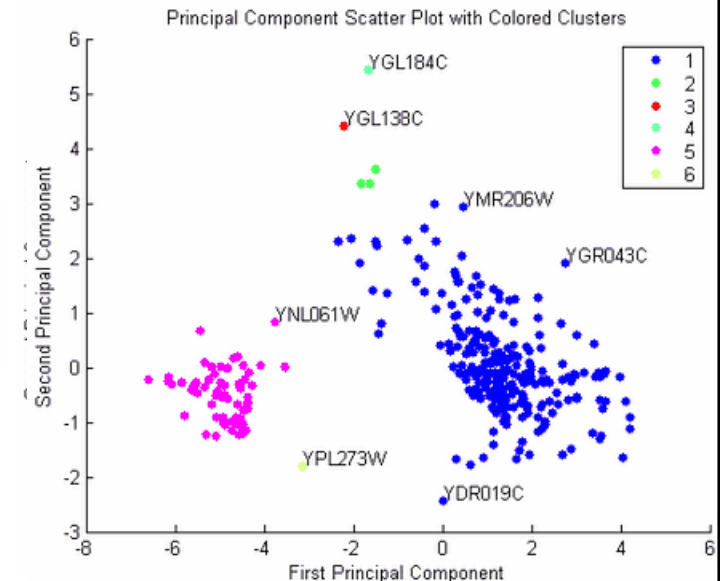
This shows that almost 90% of the variance is accounted for by the first two principal components.

PCA on Genes



MA Table	exp01	exp02	exp03	exp04	exp05	exp...	exp P
PCA-1	0.18	0.3	-0.12	-0.44	0.19	-0.39	-0.61
PCA-2	-0.16	-0.58	-0.43	-0.22	0.53	0.69	0.08
PCA-3	0.16	-0.44	-0.93	-1.23	-0.62	0.62	1.31

Yeast Microarray Data is from DeRisi, JL, Iyer, VR, and Brown, PO.(1997). "Exploring the metabolic and genetic control of gene expression on a genomic scale"; Science, Oct 24;278(5338):680-6.



MDS: Metric and Non-Metric Scaling

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Question

Given a *dissimilarity matrix* D of certain objects, can we **construct points** in k -dimensional (often 2-dimensional) space such that

Goal of metric scaling

the Euclidean distances between these points approximate the entries in the dissimilarity matrix?

Goal of non-metric scaling

the order in distances coincides with the order in the entries of the dissimilarity matrix approximately?

$$S = \sum_{i,j} (\hat{d}_{ij} - d_{ij})^2$$

Mathematically: for given k , compute points x_1, \dots, x_n in k -dimensional space such that the object function is minimized.

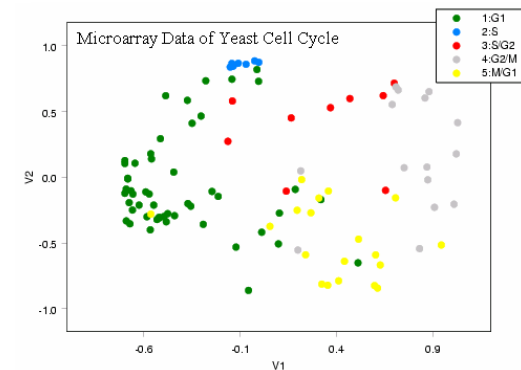
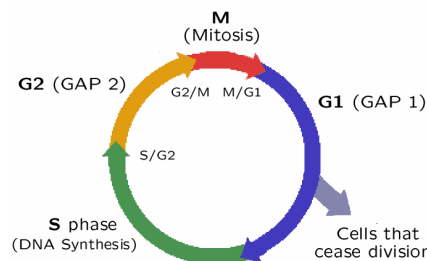
$$Stress = \sqrt{\frac{\sum_{i,j} (\hat{d}_{ij} - d_{ij})^2}{\sum_{i,j} d_{ij}^2}}$$

Microarray Data of Yeast Cell Cycle

■ Synchronized by alpha factor arrest method (Spellman et al. 1998; Chu et al. 1998)

■ 103 known genes: every 7 minutes and totally 18 time points.

■ 2D MDS Configuration Plot for 103 known genes.

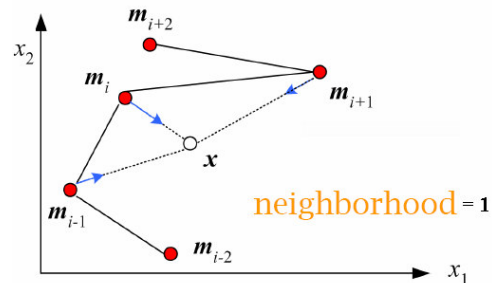


Clustering Analysis and Visualization

- ◆ Self-Organizing Maps (SOM)
- ◆ Heat Map
- ◆ Hierarchical Clustering

Self-Organizing Maps (SOM)

- SOMs were developed by **Kohonen** in the early **1980's**, original area was in the area of speech recognition.
- **Idea:** Organise data on the basis of **similarity** by putting entities **geometrically** close to each other.



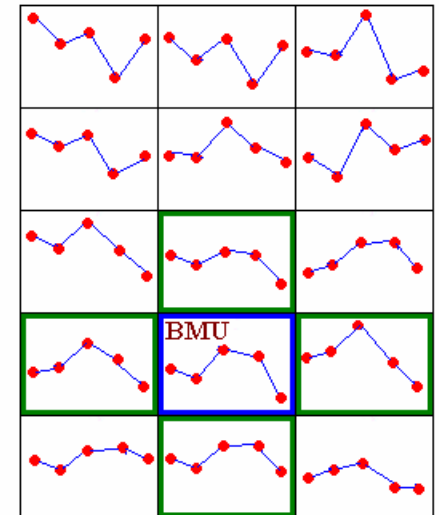
- SOM is unique in the sense that it combines both aspects. It can be used at the same time both to reduce the amount of data by **clustering**, and to construct a nonlinear projection of the data onto a **low-dimensional display**.

Step 0:
Initialize weights $w_i(t)$.
Set $\alpha(t)$ and $h_{ci}(t)$.

Learning process:

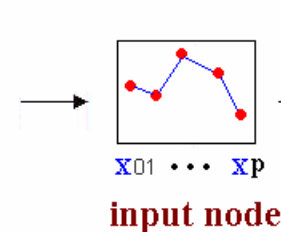
$$w_i(t+1) = \begin{cases} w_i(t) + h_{ci}(t)[x(t) - w_i(t)] & i \in N_c(t) \\ w_i(t), & \text{o.w.} \end{cases}$$

5 x 3 output node



Data Matrix

Table	X01	X02	X03	...	XP
obs 001	-0.48	-0.42	0.87		-0.35
obs 002	-0.39	-0.58	1.08		-0.58
obs 003	0.87	0.25	-0.17		-0.13
obs 004	1.57	1.03	1.22		-1.02
obs 005	-1.15	-0.86	1.21		-0.44
obs 006	0.04	-0.12	0.31		0.08
obs 007	2.95	0.45	-0.40		-0.76
obs 008	-1.22	-0.74	1.34		-0.55
obs 009	-0.73	-1.06	-0.79		0.03
obs 010	-0.58	-0.40	0.13		-0.45
obs 011	-0.50	-0.42	0.66		0.01
obs 012	-0.86	-0.29	0.42		-0.63
obs 013	-0.16	0.29	0.17		-0.04
obs ...					
obs n	-1.79	0.94	2.13		-0.66



Incrementally decrease the learning rate and the neighborhood size, and repeat

Algorithm of SOM

Step 0: Initialize weights $\mathbf{w}_i(t)$.

Set topological neighborhood parameters $N_c(t)$.

Set learning rate parameters $\alpha(t)$ and $h_{ci}(t)$.

Step 1: For each input vector $\mathbf{x}(t)$, do

a. Finding a BMU: $\|\mathbf{x}(t) - \mathbf{w}_c(t)\| = \min_i \|\mathbf{x}(t) - \mathbf{w}_i(t)\|$

b. Learning process:

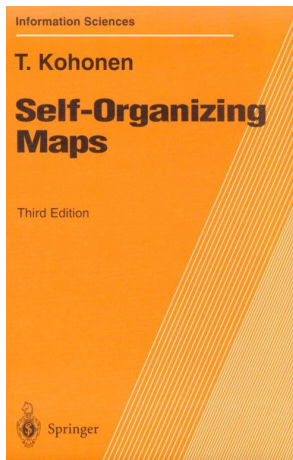
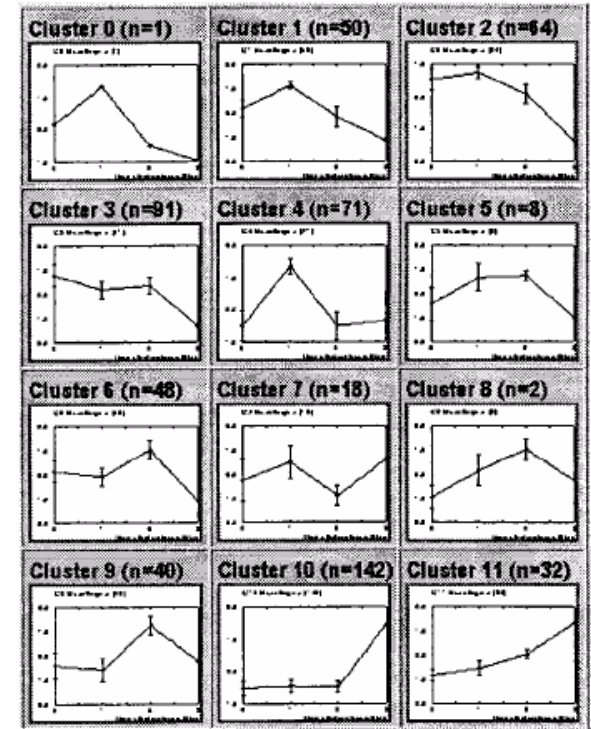
$$\mathbf{w}_i(t+1) = \begin{cases} \mathbf{w}_i(t) + h_{ci}(t)[\mathbf{x}(t) - \mathbf{w}_i(t)], & i \in N_c(t) \\ \mathbf{w}_i(t), & \text{o.w.} \end{cases}$$

c. Go to the next unvisited input vector. If there are no unvisited input vector left then go back to the very first one and go to Step 2.

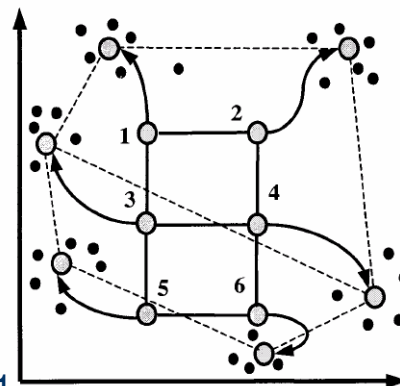
Step 2: Incrementally decrease the learning rate and the neighborhood size, and repeat Step 1.

Step 3: Keep doing Steps 1 and 2 for a sufficient number of iterations.

HL-60 4 × 3 SOM 567 genes



1995, 1997, 2001



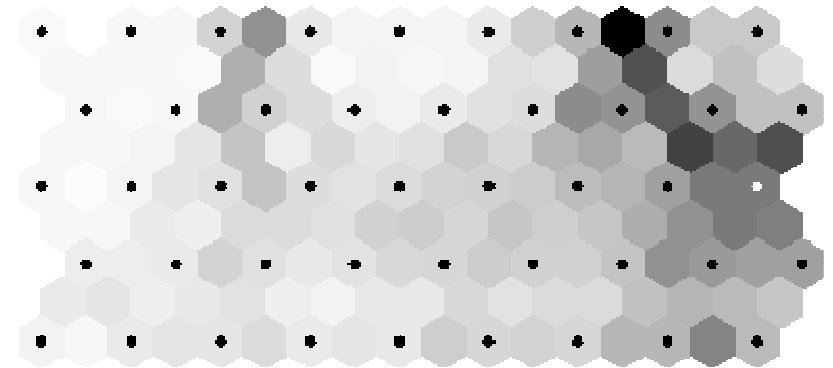
Macrophage Differentiation in HL-60 cells

Tamayo, P. et al. (1999). Interpreting patterns of gene expression with self-organizing maps: Methods and application to hematopoietic differentiation. *Proc Natl Acad Sci* 96:2907-2912.

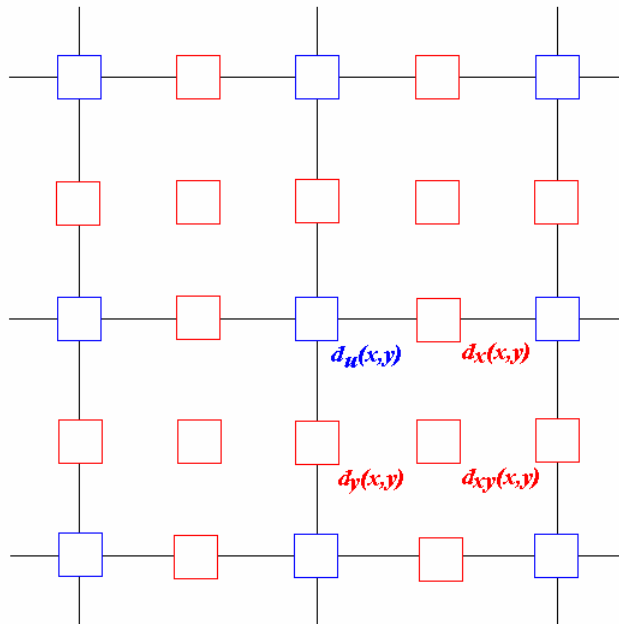
U-matrix: Unified Matrix Method

(Ultsch and Siemon 1989, Ultsch 1993)

U-matrix representation of SOM visualizes the distance between the neurons. The distance between the adjacent neurons is calculated and presented with different colorings between the adjacent nodes.



U-matrix representation of the SOM



$b(x, y)$: matrix of neurons, of size $n_x \times n_y$.

$w_i(x, y)$: matrix of weights.

$u(x, y)$: U-matrix of size $(2n_x - 1) \times (2n_y - 1)$.

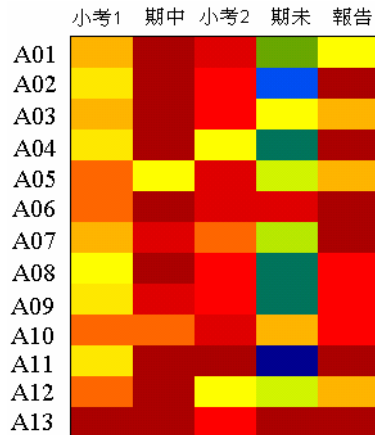
$$d_x(x, y): \|b(x, y) - b(x + 1, y)\| = \sqrt{\sum_i [w_i(x, y) - w_i(x + 1, y)]^2}$$

$$d_y(x, y): \|b(x, y) - b(x, y + 1)\| = \sqrt{\sum_i [w_i(x, y) - w_i(x, y + 1)]^2}$$

$$d_{xy}(x, y): \frac{1}{2} \left[\frac{\|b(x, y) - b(x + 1, y + 1)\|}{\sqrt{2}} + \frac{\|b(x, y + 1) - b(x + 1, y)\|}{\sqrt{2}} \right]$$

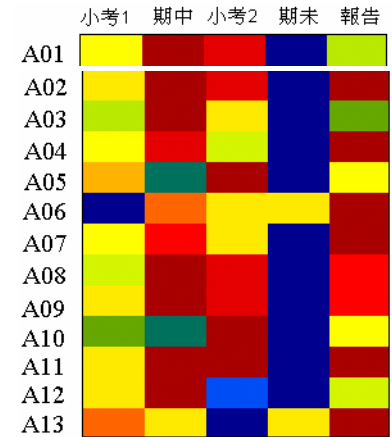
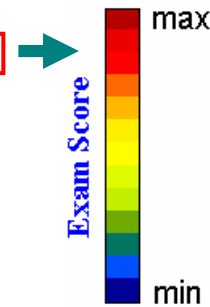
$d_u(x, y)$: the median of the surrounding elements.

Heat Map: Data Image, Matrix Visualization

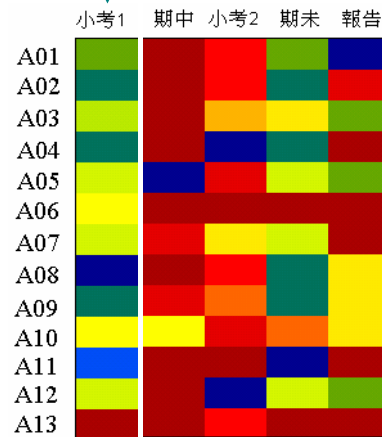


Range Matrix Condition

	A	B	C	D	E	F
1	學號	小考1	期中考	小考2	期末考	報告
2	A01	69	92	85	45	62
3	A02	66	90	83	36	90
4	A03	72	92	80	62	70
5	A04	68	90	60	37	95
6	A05	74	60	86	54	70
7	A06	77	90	88	88	95
8	A07	73	88	77	51	95
9	A08	61	90	84	40	82
10	A09	66	88	82	39	80
11	A10	76	75	87	72	80
12	A11	64	90	90	26	95
13	A12	75	90	60	55	70
14	A13	92	90	83	90	95

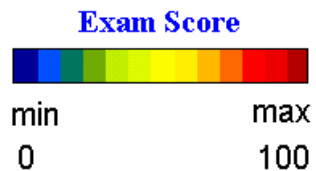


Range Row Condition



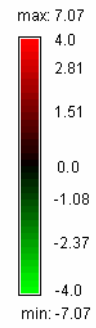
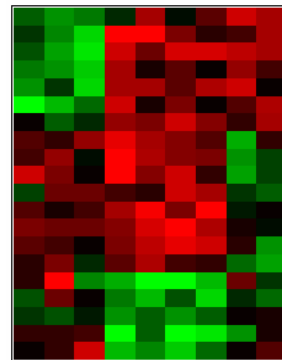
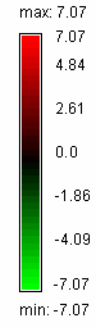
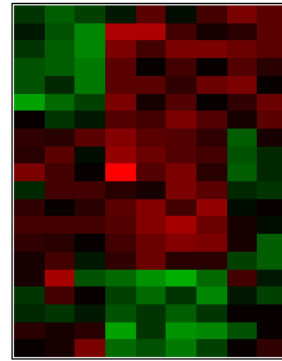
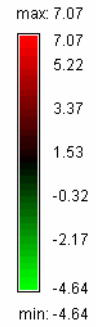
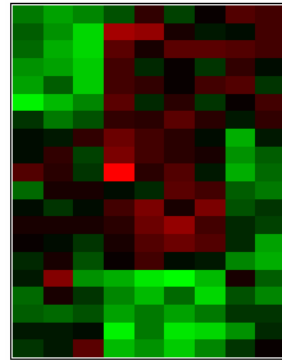
Range Column Condition

What about this one?

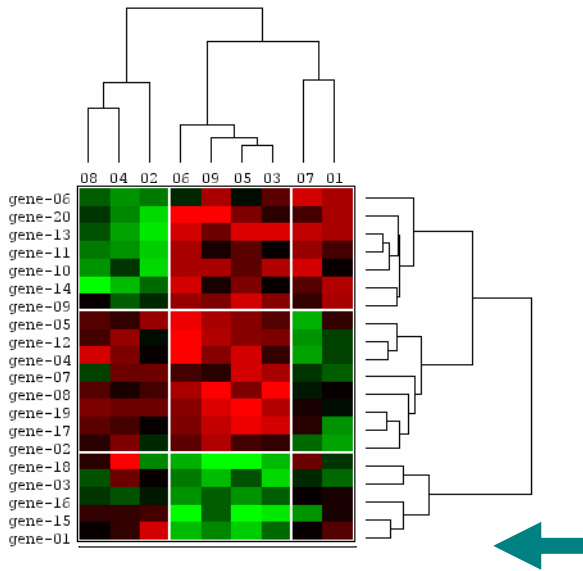


Heat Map: Display Conditions

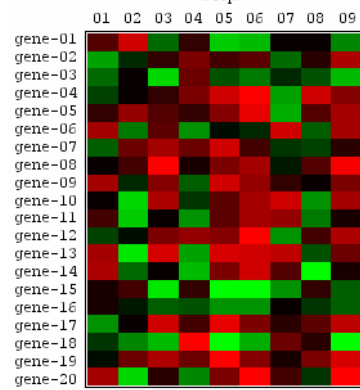
	A	B	C	D	E	F	G	H	I
1	-1.37	-2.30	-1.80	-0.55	2.45	-0.13	1.49	3.03	2.48
2	-0.68	-2.11	-3.42	4.67	4.57	1.75	0.61	0.92	2.52
3	-1.19	-2.49	-3.66	3.14	1.70	3.29	3.33	2.92	2.48
4	-1.93	-2.28	-3.16	2.51	0.32	1.49	0.21	2.20	1.03
5	-2.21	-0.79	-3.29	2.55	2.44	1.45	2.68	3.03	0.19
6	-4.14	-2.91	-1.64	3.21	0.37	1.93	0.14	1.27	2.67
7	0.21	-1.36	-0.44	2.22	1.85	3.11	2.03	0.67	2.40
8	1.13	0.79	2.25	3.65	2.52	2.09	1.13	-2.59	0.67
9	0.95	2.33	-0.07	3.89	2.72	2.13	1.75	-2.17	-0.90
10	3.04	1.85	0.21	7.07	2.01	3.05	0.76	-2.58	-1.04
11	-1.02	1.65	1.53	0.95	0.60	3.12	2.52	-0.77	-1.40
12	1.21	0.24	1.04	2.50	3.69	1.81	3.98	-0.33	0.11
13	1.74	1.60	1.70	2.02	3.45	4.46	2.69	0.41	-0.09
14	1.34	1.06	0.06	1.81	2.90	3.64	3.04	0.49	-2.33
15	0.57	1.81	-0.47	1.40	2.70	0.99	0.82	-1.61	-2.56
16	0.61	4.22	-2.03	-2.61	-4.00	-4.64	-2.92	1.55	-0.71
17	-1.13	1.64	0.01	-1.77	-2.85	-1.24	-3.41	-0.59	-1.64
18	-0.86	-1.17	-0.41	-2.20	-1.30	-2.37	-1.41	0.08	0.25
19	0.75	0.66	1.04	-4.26	-1.41	-3.99	-3.53	-2.17	0.34
20	0.15	0.68	3.18	-2.86	-2.01	-3.18	-1.58	0.10	1.28



Center Matrix Condition



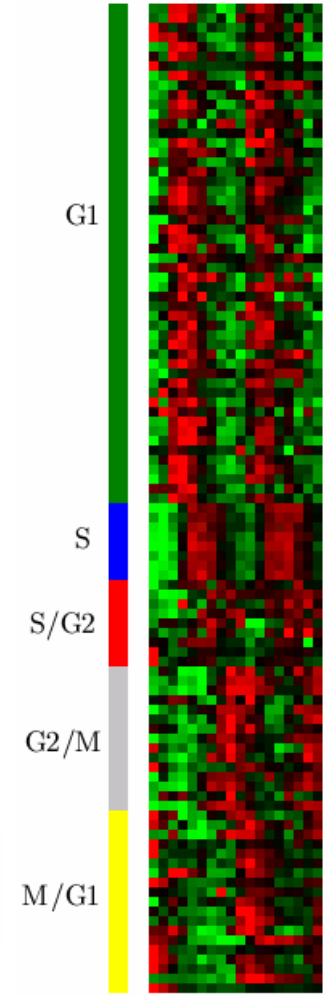
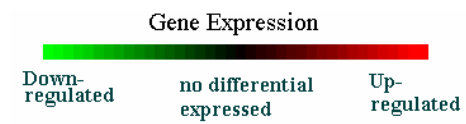
Without ordering



Microarray Data of Yeast Cell Cycle

Synchronized by alpha factor arrest method (Spellman et al. 1998; Chu et al. 1998)

103 known genes: every 7 minutes and totally 18 time points.

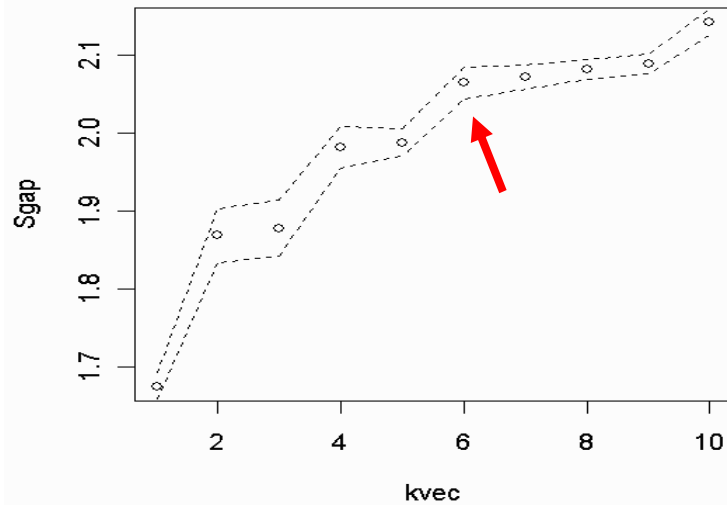


K-Means Clustering

■ Data

Baseline: Culture Medium (CM-00h)
 OH-04h, OH-12h, OH-24h
 CA-04h, CA-24h
 SO-04h, SO-24h

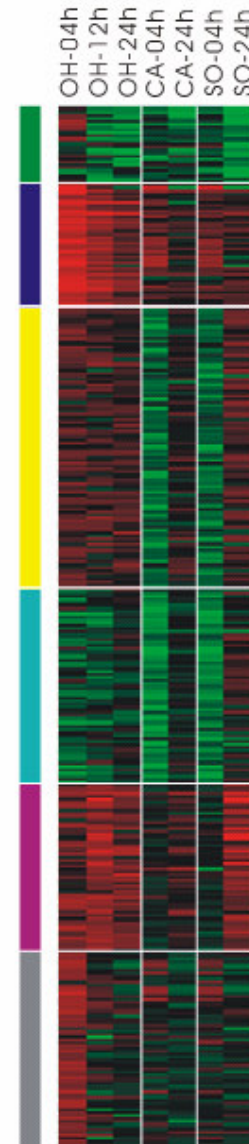
■ A set of 250 genes was selected for



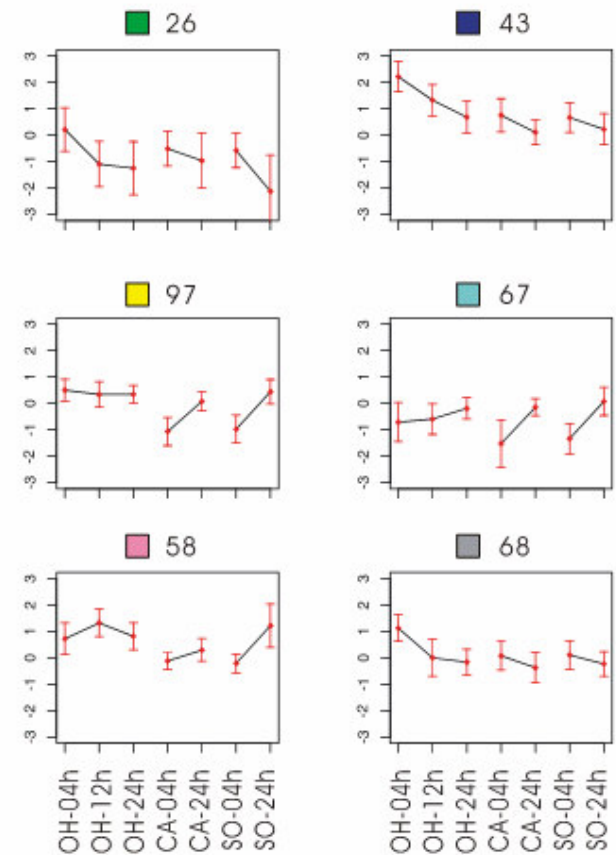
J. R. Statist. Soc. B (2001)
 63, Part 2, pp.411-423

Estimating the number of clusters in a data set via the gap statistic

Robert Tibshirani, Guenther Walther and Trevor Hastie
 Stanford University, USA



K-means Clustering



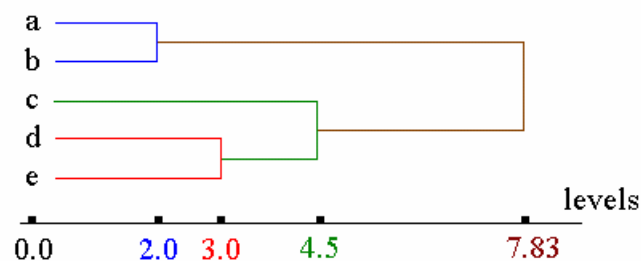
Hierarchical Clustering and Dendrogram

(Kaufman and Rousseeuw, 1990)

Example: Average-Linkage

distance matrix

	a	b	c	d	e
a	0	2	6	10	9
b		0	5	9	8
c			0	4	5
d				0	3
e					0



	{a, b}	c	d	e
{a, b}	0	5.5	9.5	8.5
c		0	4	5
d			0	3
e				0

$$D(\{a, b\}, \{c\}) = \frac{1}{2}[D(a, c) + D(b, c)]$$

$$= \frac{1}{2}(6 + 5) = 5.5$$

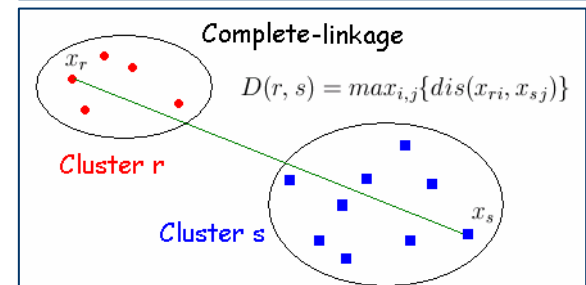
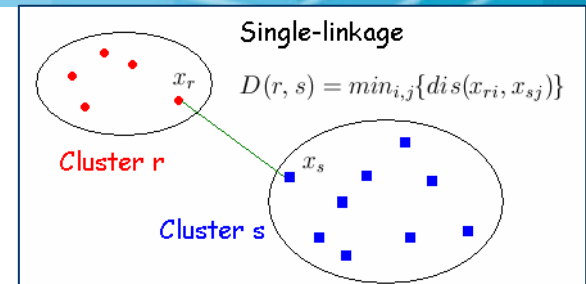
	{a, b}	c	{d, e}
{a, b}	0	5.5	9.0
c		0	4.5
{d, e}			0

$$D(\{a, b\}, \{d, e\})$$

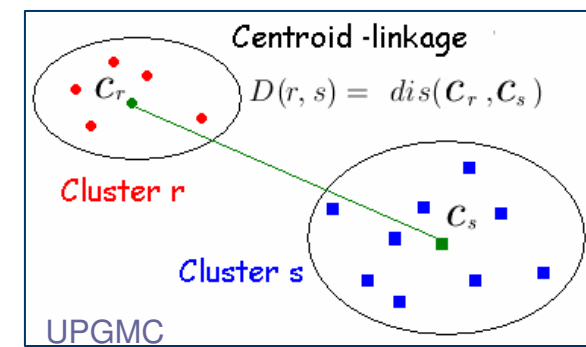
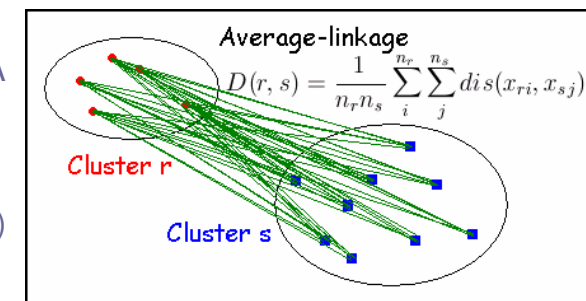
$$= \frac{1}{4}[D(a, d) + D(a, e) + D(b, d) + D(b, e)]$$

$$= \frac{1}{4}(10 + 9 + 9 + 8) = 9$$

	{a, b}	{c, d, e}
{a, b}	0	7.83
{c, d, e}		0

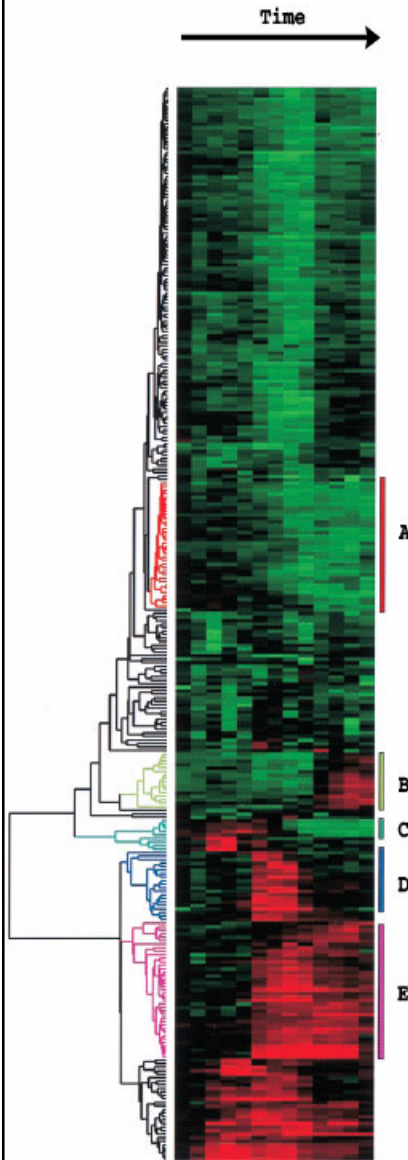


UPGMA
(Unweighted
Pair-Groups
Method
Average)



Display of Genome-Wide Expression Patterns

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Proc. Natl. Acad. Sci. USA
Vol. 95, pp. 14863–14868, December 1998
Genetics

Cluster analysis and display of genome-wide expression patterns

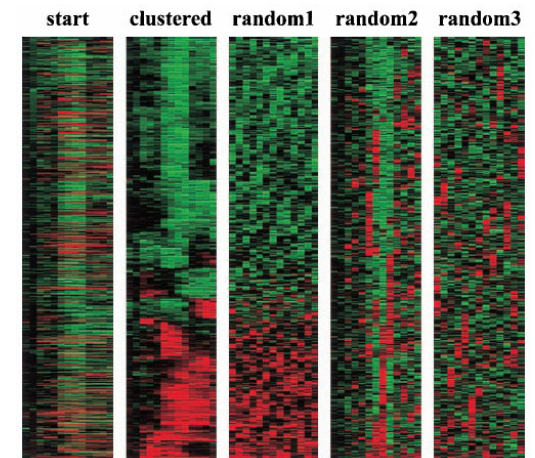
MICHAEL B. EISEN*, PAUL T. SPELLMAN*, PATRICK O. BROWN†, AND DAVID BOTSTEIN*‡

FIG. 1. Clustered display of data from time course of serum stimulation of primary human fibroblasts. Experimental details are described elsewhere (11). Briefly, foreskin fibroblasts were grown in culture and were deprived of serum for 48 hr. Serum was added back and samples taken at time 0, 15 min, 30 min, 1 hr, 2 hr, 3 hr, 4 hr, 8 hr, 12 hr, 16 hr, 20 hr, 24 hr. The final datapoint was from a separate unsynchronized sample. Data were measured by using a cDNA microarray with elements representing approximately 8,600 distinct

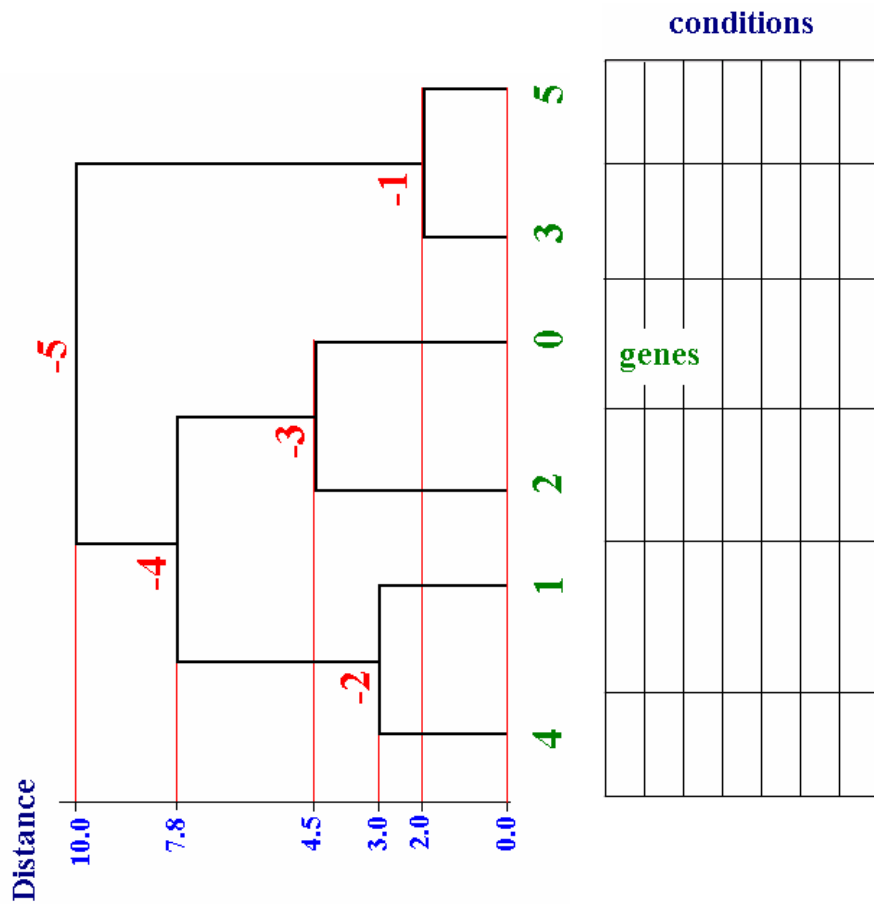
human genes. All measurements are relative to time 0. Genes were selected for this analysis if their expression level deviated from time 0 by at least a factor of 3.0 in at least 2 time points. The dendrogram and colored image were produced as described in the text; the color scale ranges from saturated green for log ratios -3.0 and below to saturated red for log ratios 3.0 and above. Each gene is represented by a single row of colored boxes; each time point is represented by a single column. Five separate clusters are indicated by colored bars and by identical coloring of the corresponding region of the dendrogram. As described in detail in ref. 11, the sequence-verified named genes in these clusters contain multiple genes involved in (A) cholesterol biosynthesis, (B) the cell cycle, (C) the immediate-early response, (D) signaling and angiogenesis, and (E) wound healing and tissue remodeling. These clusters also contain named genes not involved in these processes and numerous uncharacterized genes. A larger version of this image, with gene names, is available at <http://rana.stanford.edu/clustering/serum.html>.

Software: Cluster and TreeView

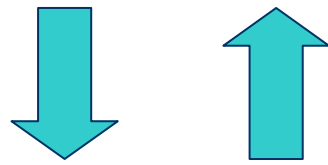
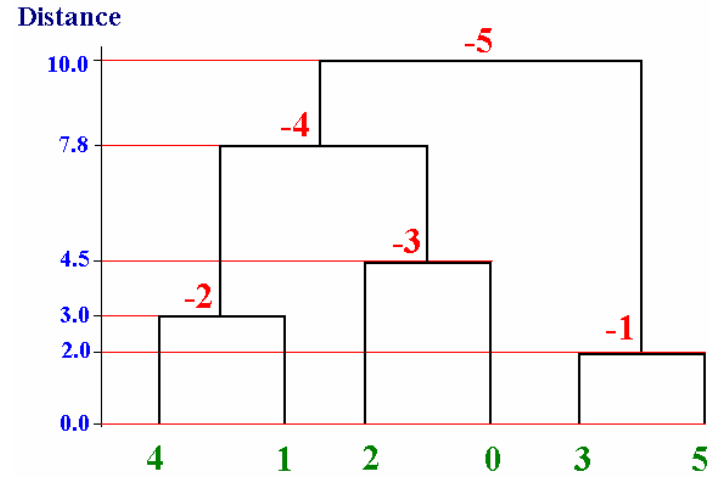
FIG. 3. To demonstrate the biological origins of patterns seen in Figs. 1 and 2, data from Fig. 1 were clustered by using methods described here before and after random permutation within rows (random 1), within columns (random 2), and both (random 3).



Dendrogram and Tree Storage



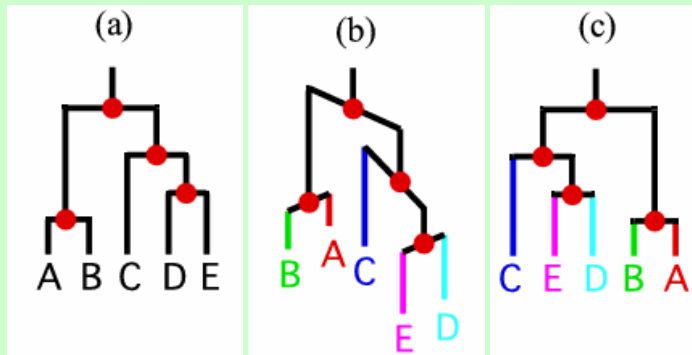
For example:
Cluster and TreeView, R



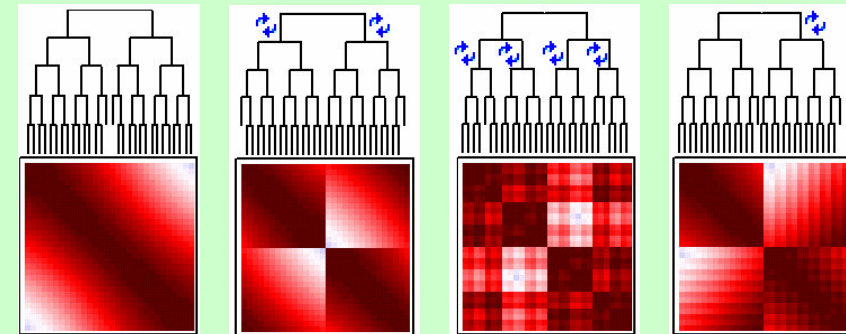
no	NodeID	Left	Right	Distance
0	-1	3	5	2
1	-2	4	1	3
2	-3	2	0	4.5
3	-4	-2	-3	7.8
4	-5	-4	-1	10

Seriation Problem for Hierarchical Clustering

Tree seriation

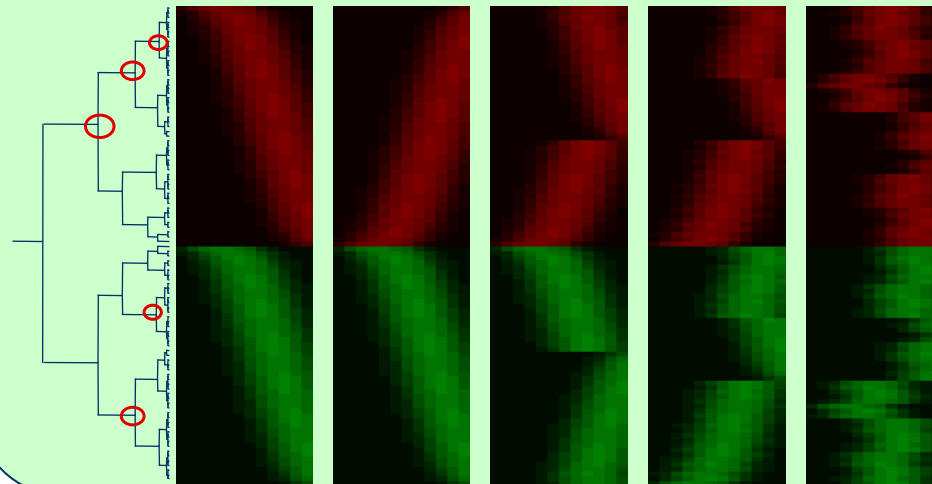


Tree seriation for proximity matrices



Tree seriation for raw data matrices

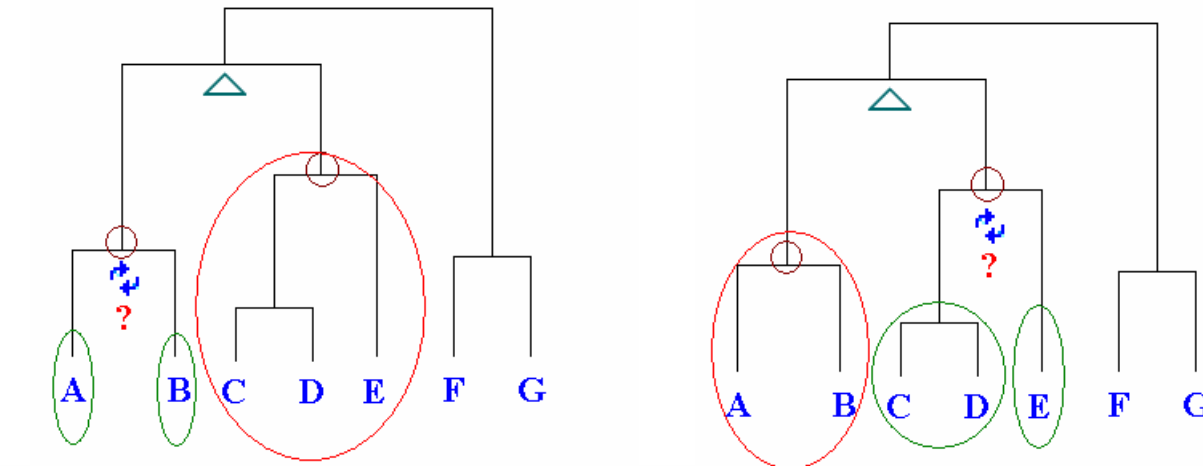
ideal model 1 flip 3 flips 5 flips many flips



Different Seriations
Generated from Identical
Tree Structure

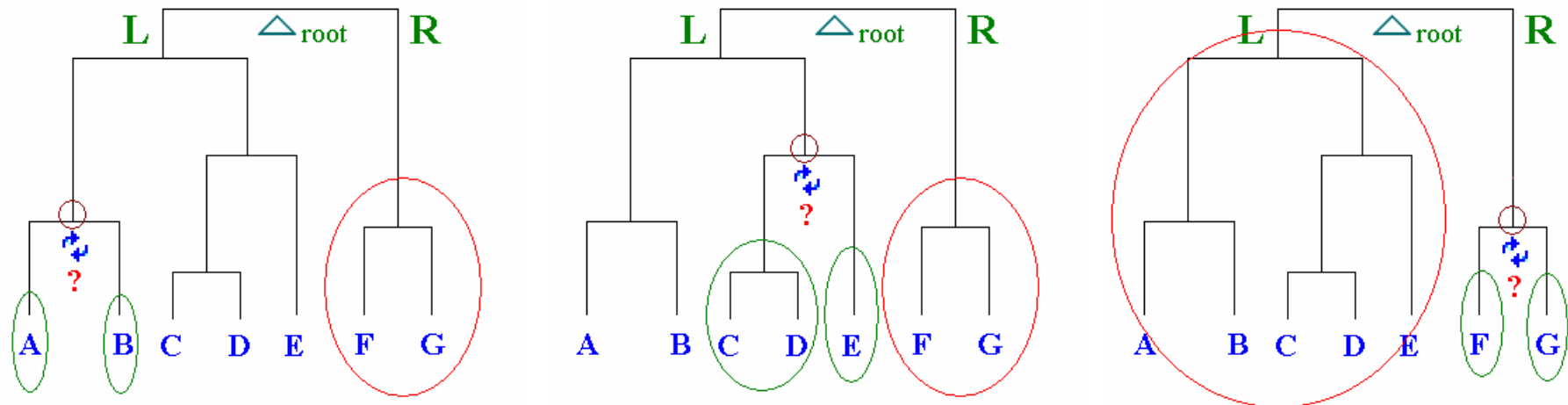
Internal Tree Flips

Uncle Approach



if $d(A, \{C, D, E\}) < d(B, \{C, D, E\})$ then flip

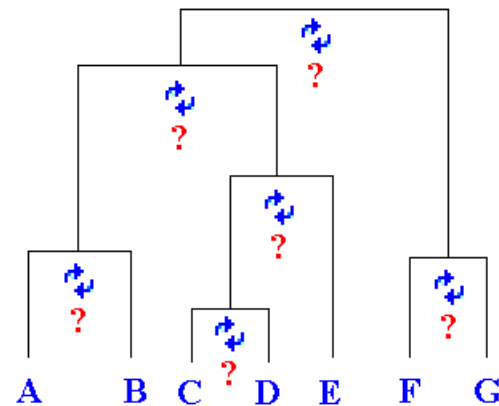
GrandPa Approach



Further reading: Ziv Bar-Joseph, David K. Gifford, and Tommi S. Jaakkola, (2001), **Fast Optimal Leaf Ordering** for Hierarchical Clustering. *Bioinformatics* 17(Suppl. 1):S22–S29.

External Tree Flips

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External Ordering

D E A F B C G

As match as possible

How to build an external ordering?

- (1) Based on average expression level (Cluster Software, Eisen et al 1998)
- (2) Using the results of a one-dimensional SOM
- (3) ...

Further reading: Tien, Y. J., Lee, Y. S, Wu, H. M. and Chen, C. H. (2006) Integration of clustering and visualization methods for simultaneously identifying coherent local clusters with smooth global patterns in gene expression profiles.

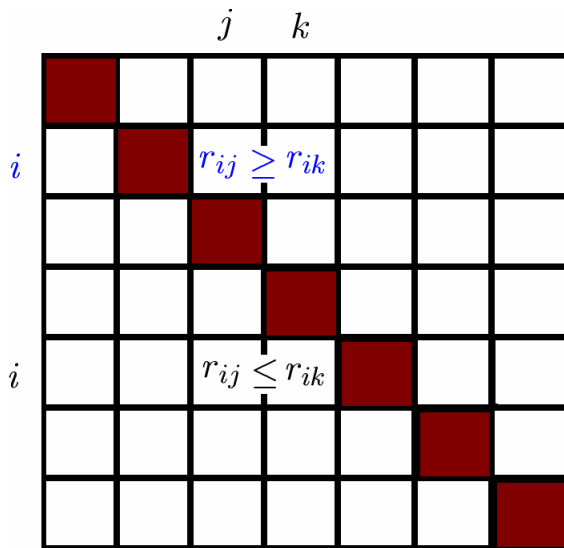
Criteria for a “good” Permutation

When T is symmetric, we usually want T' to approximate a Robinson form (Robinson (1951)).

Global/local Criterion: Anti-Robinson Measurements

permuted matrix, $D = [d_{ij}]$

Robinson Form



$$AR(i) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) + \sum_{i < j < k} I(d_{ij} > d_{ik}) \right],$$

$$AR(s) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) \cdot |d_{ij} - d_{ik}| + \sum_{i < j < k} I(d_{ij} > d_{ik}) \cdot |d_{ij} - d_{ik}| \right],$$

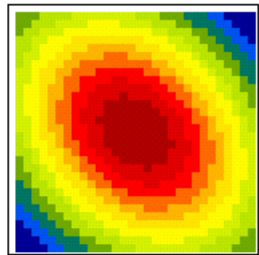
$$AR(w) = \sum_{i=1}^p \left[\sum_{j < k < i} I(d_{ij} < d_{ik}) |j - k| |d_{ij} - d_{ik}| + \sum_{i < j < k} I(d_{ij} > d_{ik}) |j - k| |d_{ij} - d_{ik}| \right].$$

$r_{ij} \leq r_{ik}$ if $j < k < i$, $r_{ij} \geq r_{ik}$ if $i < j < k$

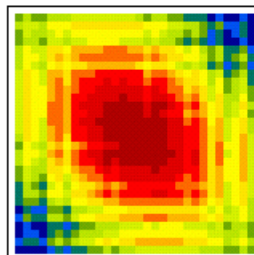


Min.

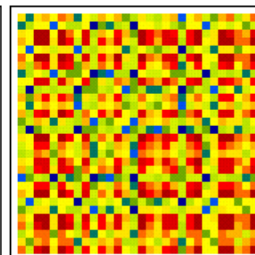
Max.



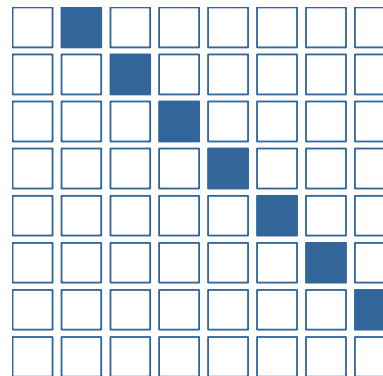
Robinson



pre-Robinson



Local criterion: Minimal Span Loss Function



$$MS = \sum_{i=1}^{n-1} d_{i,i+1}$$

Further Reading

Michael Friendly, Ernest Kwan, (2003) Effect ordering for data displays, Computational Statistics & Data Analysis, v.43 n.4, p.509-539.

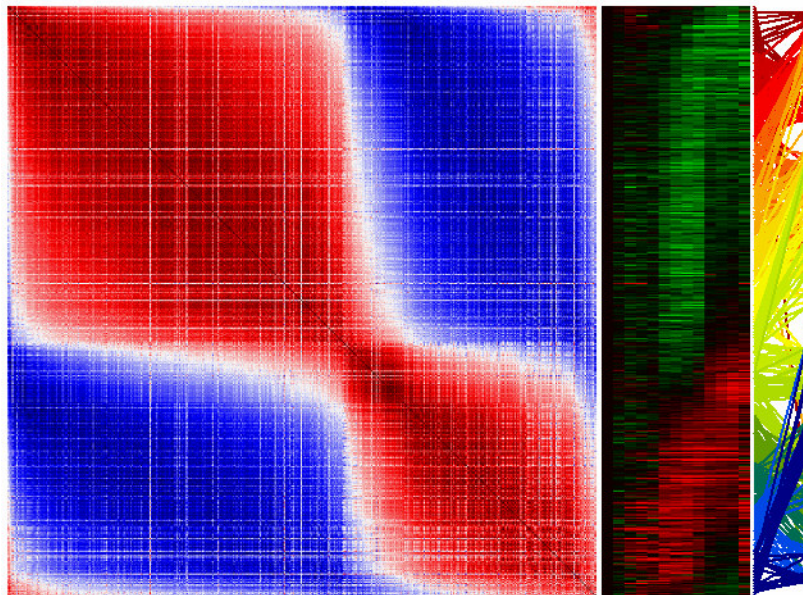
Global vs Local Seriation

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GAP Elliptical Seriation

An algorithm for identifying global clustering patterns and smoothing temporal expression profiles

GAP Elliptical Seriation



Michael Eisen Tree Seriation

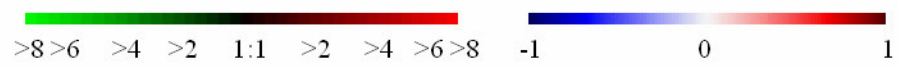
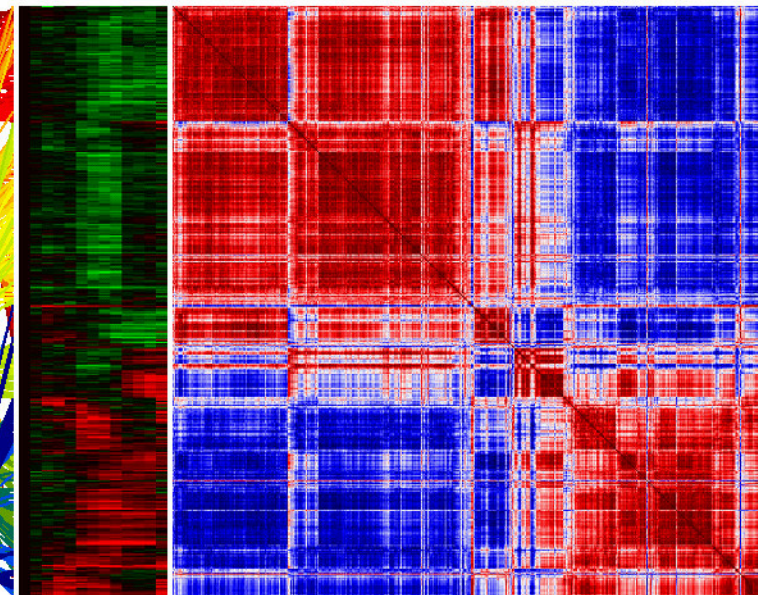
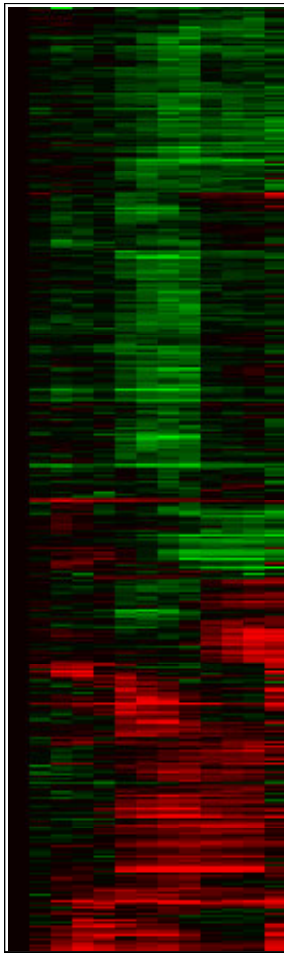


Image source: Dr. Chen Chun-houh's slide

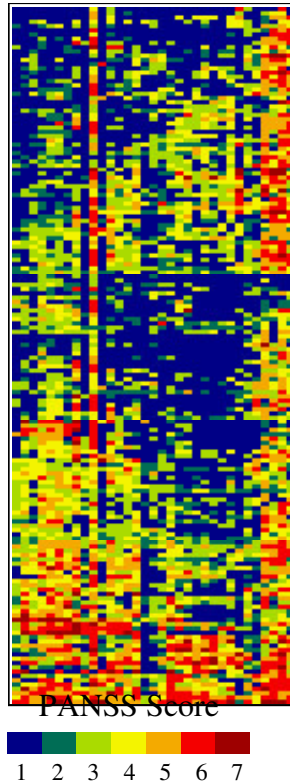
Visualization of Data Matrices

Simple ← Information Visualization of Data Matrices → **Difficult**

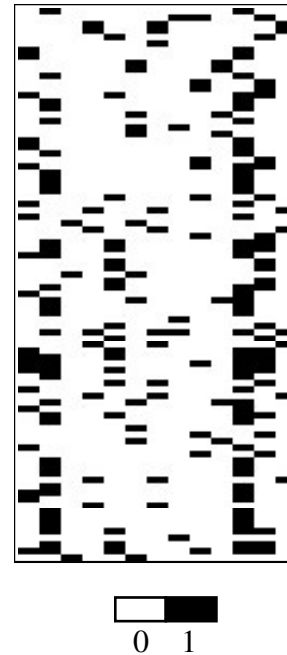
Continuous
(Gene/Time)



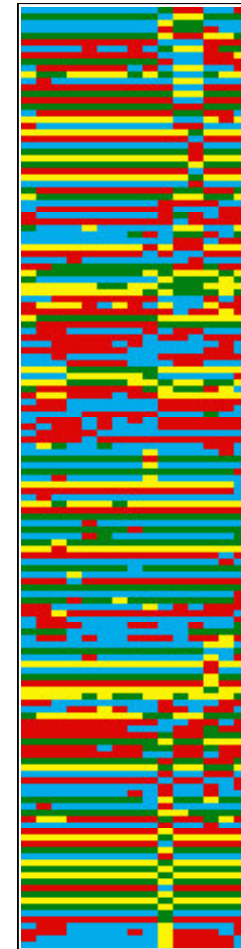
Ordinal
(Patient/Symptom)



Binary
(Mouse/Tumor)



Categorical
(Subject/SNP)



>8 >6 >4 >2 1:1 >2 >4 >6 >8 Log2ratio

Image source: Chen Chun-houh's slide

A C G T

Cluster Validation

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Assess the **quality** and **reliability** of the cluster sets.

- **Quality:** clusters can be measured in terms of **homogeneity** and **separation**.
- **Reliability:** cluster structure is not formed by chance.
- **Ground Truth:** from domain knowledge.

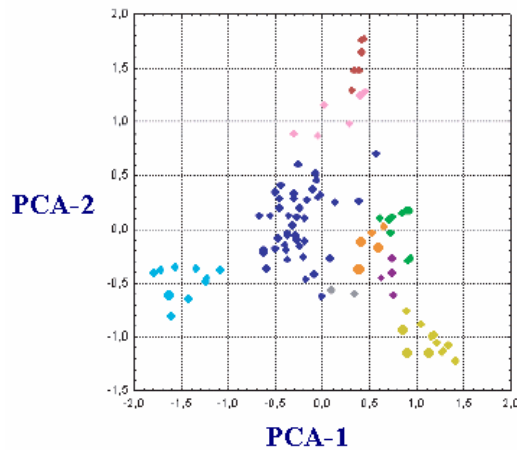
NOTE:

Help to decide the **number of clusters in the data**.

Choosing the Number of Clusters

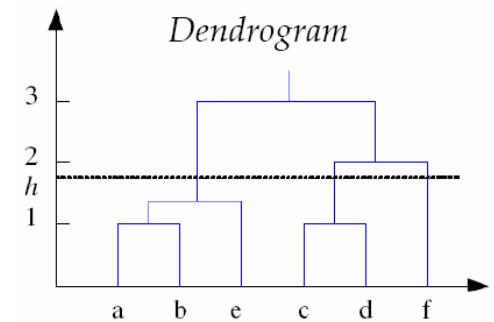
(1) K is defined by the application.

(2) Plot the data in two PAC dimensions.



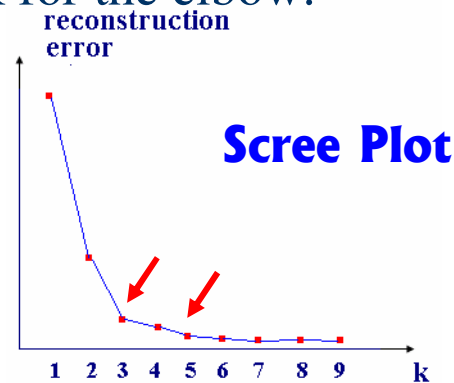
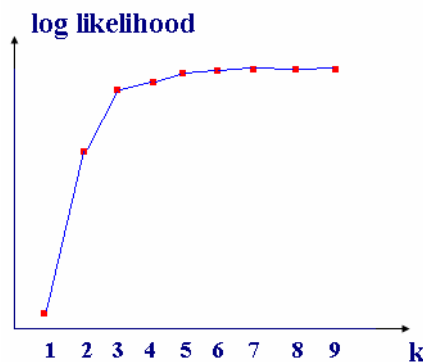
(e.g., k-means:
within-cluster sum of squares)

(4) Hierarchical clustering:
look at the difference between levels in the tree.



Calinski and Harabasz (1974): $CH(k)$
Hartigan (1975): $H(k)$
Krzanowski and Lai (1985): $KL(k)$
Kaufman and Rousseeuw (1990): $s(i)$

(3) Plot the **reconstruction error** or log likelihood as a function of k, and look for the elbow.



J. R. Statist. Soc. B (2001)
63, Part 2, pp. 411–423
Estimating the number of clusters in a data set via the gap statistic
Robert Tibshirani, Guenther Walther and Trevor Hastie
Stanford University, USA

Literatures on Cluster Validation

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- Francisco R. Pinto, João A. Carriço, Mário Ramirez and Jonas S Almeida, (2007), Ranked Adjusted Rand: integrating distance and partition information in a measure of clustering agreement, BMC Bioinformatics, 8:44.

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More than 30 papers for Microarray!

with? BMC

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Cluster Validation Index

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Internal Measures

Stability Measures

Comparing Partitions

Biological Measures

Cluster Validation

Validation Index

- Internal Measures
- Stability Measures
- Comparing Partitions
- Biological Measures

Internal Measures

- (1) Dunn Index
- (2) Within Cluster Variance
- (3) Silhouette Width
- (4) Connectivity

Stability Measures

- (1) APN: Average Proportion of Non-overlap
- (2) AD: Average Distance
- (3) ADM: Average Distance between Means
- (4) FOM: Figure of Merit

Comparing Partitions

- (1) Rand Index
- (2) Adjusted Rand Index
- (3) Jaccard Coefficient
- (4) Minkowski Index

LungMarkerGene_68x144-marker.txt

Biological Measures

- (1) BHI: Biological Homogeneity Index
- (2) BSI: Biological Stability Index

LungMarkerGene_68x144-marker.txt

Close Report

See also

clValid: an R package for cluster validation.

Statistical Evaluation: Internal Measures

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Compactness

Homogeneity

Separation

Connectivity

$$Conn(\mathcal{C}) = \sum_{i=1}^N \sum_{j=1}^L d_{i,nn_{i(j)}}$$

$$d_{i,nn_{i(j)}} = \begin{cases} 0, & \text{for } i \text{ and } j \text{ are in the same cluster,} \\ 1/j, & \text{otherwise.} \end{cases}$$

Dunn index (Dunn, 1974)

$$D(\mathcal{C}) = \frac{\min_{C_k, C_l \in \mathcal{C}, C_k \neq C_l} \left(\min_{i \in C_k, j \in C_l} \text{dist}(i, j) \right)}{\max_{C_m \in \mathcal{C}} \text{diam}(C_m)}$$

Within-cluster Variance

$$V(\mathcal{C}) = \sqrt{\frac{1}{N} \sum_{C_k \in \mathcal{C}} \sum_{i \in C_k} \text{dist}(i, \mu_k)}$$

Silhouette Width (Rousseeuw, 1987)

$$S(\mathcal{C}) = \sum_{i=1}^N \frac{S(i)}{N}, \quad S(i) = \frac{b_i - a_i}{\max(b_i, a_i)}$$

the average distance between i and the observations in the closet other cluster

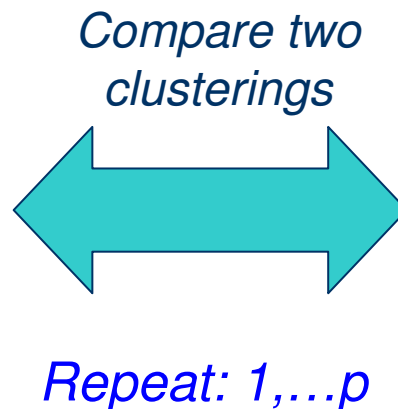
the average distance between i and all other observations in the same cluster.

Statistical Evaluation: Stability

- Average Proportion of Non-overlap (APN)
- Average Distance (AD)
- Average Distance between Means (ADM)
- Prediction Strength: Figure of Merit (FOM)

	A	B	C	D	E	F	G	H	I
1	-1.37	-2.30	-1.80	-0.55	2.45	-0.13	1.49	3.03	2.48
2	-0.68	-2.11	-3.42	4.67	4.57	1.75	0.61	0.92	2.52
3	-1.19	-2.49	-3.66	3.14	1.70	3.29	3.33	2.92	2.48
4	-1.93	-2.28	-3.16	2.51	0.32	1.49	0.21	2.20	1.03
5	-2.21	-0.79	-3.29	2.55	2.44	1.45	2.68	3.03	0.18
6	-4.14	-2.91	-1.64	3.21	0.37	1.93	0.14	1.27	2.67
7	0.21	-1.36	-0.44	2.22	1.85	3.11	2.03	0.67	2.40
8	1.13	0.79	2.25	3.65	2.52	2.09	1.13	-2.59	0.67
9	0.95	2.33	-0.07	3.89	2.72	2.13	1.75	-2.17	-0.90
10	3.04	1.85	0.21	7.07	2.01	3.05	0.76	-2.58	-1.04
11	-1.02	1.65	1.53	0.95	0.60	3.12	2.52	-0.77	-1.40
12	1.21	0.24	1.04	2.50	3.69	1.81	3.98	-0.33	0.11
13	1.74	1.60	1.70	2.02	3.45	4.46	2.69	0.41	-0.09
14	1.34	1.06	0.06	1.81	2.90	3.64	3.04	0.49	-2.33
15	0.57	1.81	-0.47	1.40	2.70	0.99	0.82	-1.61	-2.56
16	0.61	4.22	-2.03	-2.61	-4.00	-4.64	-2.92	1.55	-0.71
17	-1.13	1.64	0.01	-1.77	-2.85	-1.24	-3.41	-0.59	-1.64
18	-0.86	-1.17	-0.41	-2.20	-1.30	-2.37	-1.41	0.08	0.25
19	0.75	0.66	1.04	-4.26	-1.41	-3.99	-3.53	-2.17	0.34
20	0.15	0.68	3.18	-2.86	-2.01	-3.18	-1.58	0.10	1.28

Full data ($n \times p$)



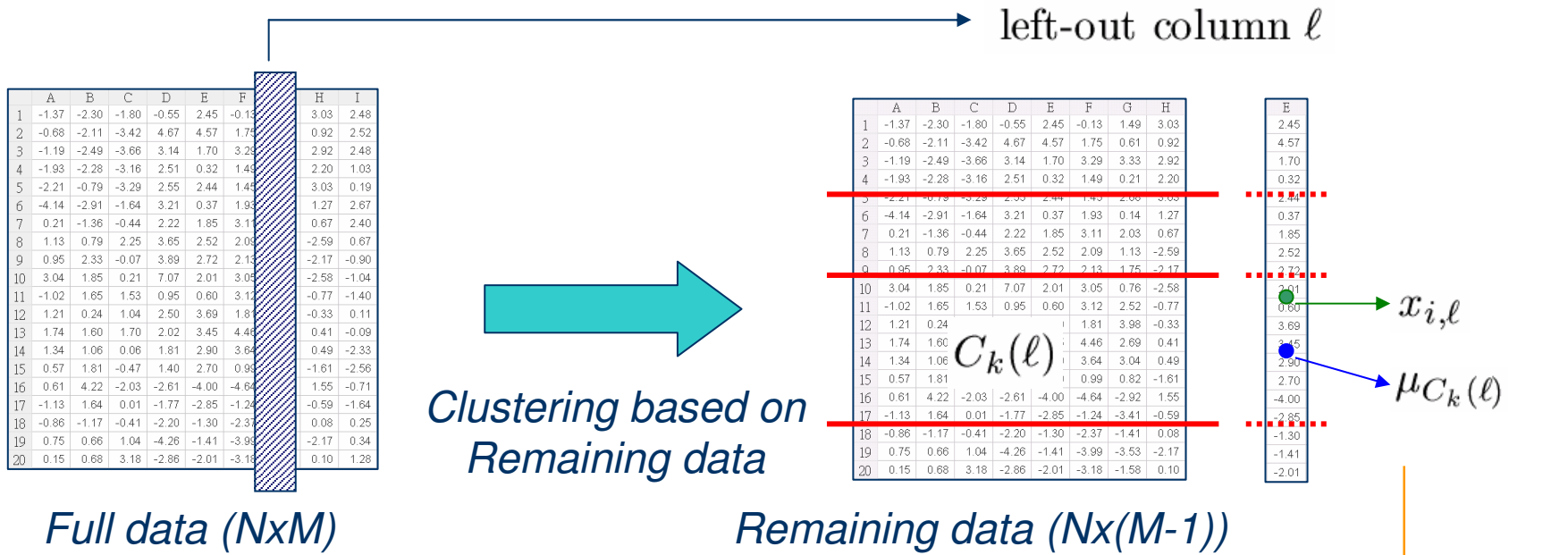
	A	B	C	D	E	F	G	H
1	-1.37	-2.30	-1.80	-0.55	2.45	-0.13	1.49	3.03
2	-0.68	-2.11	-3.42	4.67	4.57	1.75	0.61	0.92
3	-1.19	-2.49	-3.66	3.14	1.70	3.29	3.33	2.92
4	-1.93	-2.28	-3.16	2.51	0.32	1.49	0.21	2.20
5	-2.21	-0.79	-3.29	2.55	2.44	1.45	2.68	3.03
6	-4.14	-2.91	-1.64	3.21	0.37	1.93	0.14	1.27
7	0.21	-1.36	-0.44	2.22	1.85	3.11	2.03	0.67
8	1.13	0.79	2.25	3.65	2.52	2.09	1.13	-2.59
9	0.95	2.33	-0.07	3.89	2.72	2.13	1.75	-2.17
10	3.04	1.85	0.21	7.07	2.01	3.05	0.76	-2.58
11	-1.02	1.65	1.53	0.95	0.60	3.12	2.52	-0.77
12	1.21	0.24	1.04	2.50	3.69	1.81	3.98	-0.33
13	1.74	1.60	1.70	2.02	3.45	4.46	2.69	0.41
14	1.34	1.06	0.06	1.81	2.90	3.64	3.04	0.49
15	0.57	1.81	-0.47	1.40	2.70	0.99	0.82	-1.61
16	0.61	4.22	-2.03	-2.61	-4.00	-4.64	-2.92	1.55
17	-1.13	1.64	0.01	-1.77	-2.85	-1.24	-3.41	-0.59
18	-0.86	-1.17	-0.41	-2.20	-1.30	-2.37	-1.41	0.08
19	0.75	0.66	1.04	-4.26	-1.41	-3.99	-3.53	-2.17
20	0.15	0.68	3.18	-2.86	-2.01	-3.18	-1.58	0.10

Remaining data ($n \times (p-1)$)

left-out column ℓ

sample

Figure of Merit (FOM)



$$FOM(\ell, C) = \sqrt{\frac{1}{N} \sum_{k=1}^K \sum_{i \in C_k(\ell)} \text{dist}(x_{i,\ell}, \mu_{C_k(\ell)})} \times \sqrt{\frac{N}{N-K}}$$

$$FOM(C) = \frac{1}{M} \sum_{\ell=1}^M FOM(\ell, C)$$

K. Y. Yeung, D. R. Haynor and W. L. Ruzzo, (2001), Validating clustering for gene expression data, *Bioinformatics* 17(4), 309-318.

Agreement with Reference Partition

- Rand index
- Jaccard coefficient
- Minikowski Measure
- Adjusted Rand index

- Rand index, [0, 1], maximum:

$$R(\mathcal{U}, \mathcal{V}) = \frac{n_{11} + n_{00}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

- Jaccard coefficient, [0, 1], maximum:

$$J(\mathcal{U}, \mathcal{V}) = \frac{n_{11}}{n_{11} + n_{10} + n_{01}}$$

- Minikowski measure:

$$M(\mathcal{U}, \mathcal{V}) = \sqrt{\frac{n_{10} + n_{01}}{n_{11} + n_{01}}}$$

$$\mathcal{U} = \{U_1, \dots, U_K\}$$



$$A = \left[\begin{array}{c|ccc} & O_1 & O_1 & \dots & O_n \\ \hline O_1 & & & & \\ O_2 & & & & \\ \dots & & & & \\ O_n & & & & \end{array} \right]$$

$$A_{ij} = \mathbb{I}(O_i \in U_k, O_j \in U_k)$$

$$\mathcal{V} = \{V_1, \dots, V_S\}$$



$$B = \left[\begin{array}{c|ccc} & O_1 & O_1 & \dots & O_n \\ \hline O_1 & & & & \\ O_2 & & & & \\ \dots & & & & \\ O_n & & & & \end{array} \right]$$

$$B_{ij} = \mathbb{I}(O_i \in V_s, O_j \in V_s)$$



$$n_{11} = \#\{(O_i, O_j); \mathbb{I}(A_{ij} = 1, B_{ij} = 1)\}$$

$$n_{10} = \#\{(O_i, O_j); \mathbb{I}(A_{ij} = 1, B_{ij} = 0)\}$$

$$n_{01} = \#\{(O_i, O_j); \mathbb{I}(A_{ij} = 0, B_{ij} = 1)\}$$

$$n_{00} = \#\{(O_i, O_j); \mathbb{I}(A_{ij} = 0, B_{ij} = 0)\}$$



Adjusted Rand index

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May be the most widely used Cluster Validation Index!

$$A = \begin{array}{c|cccc|c} \text{Cluster} & V_1 & V_1 & \cdots & V_C & \text{Sums} \\ \hline U_1 & n_{11} & n_{12} & & n_{1C} & n_{1\cdot} \\ U_2 & n_{21} & n_{22} & & n_{2C} & n_{2\cdot} \\ \cdots & & & & & \cdots \\ U_R & n_{R1} & n_{R2} & & n_{RC} & n_{R\cdot} \\ \hline \text{Sums} & n_{\cdot 1} & n_{\cdot 1} & \cdots & n_{\cdot C} & n \end{array}$$

$$R(\mathcal{U}, \mathcal{V}) = 1 + \frac{\sum_{i=1}^R \sum_{j=1}^C n_{ij}^2 - \frac{1}{2}(\sum_{i=1}^R n_{i\cdot}^2 + \sum_{j=1}^C n_{\cdot j}^2)}{\binom{n}{2}}$$

$$R(\mathcal{U}, \mathcal{V})_{adj} = \frac{\sum_{i=1}^R \sum_{j=1}^C \binom{n_{ij}}{2} - \sum_{i=1}^R \sum_{j=1}^C \binom{n_{i\cdot}}{2} \binom{n_{\cdot j}}{2} / \binom{n}{2}}{\frac{1}{2}[\sum_{i=1}^R \binom{n_{i\cdot}}{2} + \sum_{j=1}^C \binom{n_{\cdot j}}{2}] - \sum_{i=1}^R \sum_{j=1}^C \binom{n_{i\cdot}}{2} \binom{n_{\cdot j}}{2} / \binom{n}{2}}$$

Lawrence Hubert and Phipps Arabie (1985), Comparing partitions, Journal of Classification 2(1), 193-218.

Biological Evaluation

- Biological Homogeneity Index (BHI)
- Biological Stability Index (BSI)

Example:
GO (Gene Ontology)
Multiple Functional Categories

ProbeSet	Clustering	GO-BP Category
38389_at	1	0
1662_r_at	1	0
32607_at	1	0
1582_at	1	0
34699_at	1	0
37890_at	2	0
36008_at	2	1 2 3
36591_at	2	1 2 3 8 10
32081_at	2	1 2 3 4 5 6 7 9 10
668_s_at	2	1 2 3
41535_at	2	1 2 3 4
37666_at	2	1 2 3
40310_at	2	1 2 3 4 5 8 9
34256_at	3	1 2 3
38790_at	3	1
39175_at	3	1 2 3
35819_at	3	1 8
37639_at	3	1 2 3
31508_at	3	1 9
31505_at	4	1 2 3
1882_g_at	4	1 2 3 4 6
33154_at	4	1 2 3
837_s_at	4	1 2 3
35194_at	4	1
38422_s_at	4	1 2 3 4 5
33131_at	4	1 2 3 4 6 7

Susmita Datta and Somnath Datta, (2006), Methods for evaluating clustering algorithms for gene expression data using a reference set of functional classes, BMC Bioinformatics 7:397.

Biological Evaluation: Homogeneity

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Biological Homogeneity Index (BHI)

- $\mathcal{B} = \{B_1, \dots, B_F\}$: a set of F functional classes, not necessarily disjoint,
- B^i : the functional class containing gene i (with possibly more than one functional class containing i).
- B^j : the function class containing gene j ,
- $I(B^i = B^j) = \begin{cases} 1, & \text{if } B^i \text{ and } B^j \text{ match,} \\ 0, & \text{otherwise.} \end{cases}$
- Given statistical clustering partition $\mathcal{C} = \{C_1, \dots, C_K\}$ and set of biological classes $\mathcal{B} = \{B_1, \dots, B_F\}$, the BHI is defined as

$$BHI(\mathcal{C}, \mathcal{B}) = \frac{1}{K} \sum_{k=1}^K \frac{1}{n_k(n_k - 1)} \sum_{i \neq j; i, j \in C_k} I(B^i = B^j) .$$

- $n_k = n(C_k \cap \mathcal{B})$: the number of annotated genes in statistical cluster C_k .
- Range: $[0, 1]$, maximum.

Biological Evaluation: Stability

Biological Stability Index (BSI)

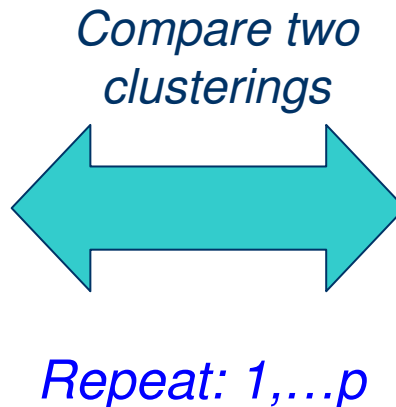
- The BSI is defined as

$$BSI(C, B) = \frac{1}{F} \sum_{k=1}^F \frac{1}{n(B_k)(n(B_k) - 1)} \frac{1}{M} \sum_{\ell=1}^M \sum_{i \neq j; i, j \in B_k} \frac{n(C^{i,0} \cap C^{j,\ell})}{n(C^{i,0})},$$

- $C^{i,0}$: the statistical cluster containing observation i based on all the data.
- $C^{j,\ell}$: the statistical cluster containing observation j when column ℓ is removed.
- Range $[0, 1]$: maximum.

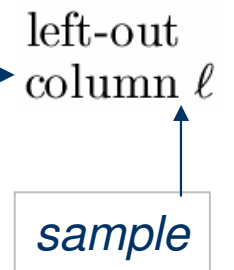
	A	B	C	D	E	F	G	H	I
1	-1.37	-2.30	-1.80	-0.55	2.45	-0.13	1.49	3.03	2.48
2	-0.68	-2.11	-3.42	4.67	4.57	1.75	0.61	0.92	2.52
3	-1.19	-2.49	-3.66	3.14	1.70	3.29	3.33	2.92	2.48
4	-1.93	-2.28	-3.16	2.51	0.32	1.49	0.21	2.20	1.03
5	-2.21	-0.79	-3.29	2.55	2.44	1.45	2.68	3.03	0.92
6	-4.14	-2.91	-1.64	3.21	0.37	1.93	0.14	1.27	2.67
7	0.21	-1.36	-0.44	2.22	1.85	3.11	2.03	0.67	2.40
8	1.13	0.79	2.25	3.65	2.52	2.09	1.13	-2.59	0.67
9	0.95	2.33	-0.07	3.89	2.72	2.13	1.75	-2.17	-0.90
10	3.04	1.85	0.21	7.07	2.01	3.05	0.76	-2.58	-1.04
11	-1.02	1.65	1.53	0.95	0.60	3.12	2.52	-0.77	-1.40
12	1.21	0.24	1.04	2.50	3.69	1.81	3.98	-0.33	0.11
13	1.74	1.60	1.70	2.02	3.45	4.46	2.69	0.41	-0.09
14	1.34	1.06	0.06	1.81	2.90	3.64	3.04	0.49	-2.33
15	0.57	1.81	-0.47	1.40	2.70	0.99	0.82	-1.61	-2.56
16	0.61	4.22	-2.03	-2.61	-4.00	-4.64	-2.92	1.55	-0.71
17	-1.13	1.64	0.01	-1.77	-2.85	-1.24	-3.41	-0.59	-1.64
18	-0.86	-1.17	-0.41	-2.20	-1.30	-2.37	-1.41	0.08	0.25
19	0.75	0.66	1.04	-4.26	-1.41	-3.99	-3.53	-2.17	0.34
20	0.15	0.68	3.18	-2.86	-2.01	-3.18	-1.58	0.10	1.28

Full data ($n \times p$)



	A	B	C	D	E	F	G	H
1	-1.37	-2.30	-1.80	-0.55	2.45	-0.13	1.49	3.03
2	-0.68	-2.11	-3.42	4.67	4.57	1.75	0.61	0.92
3	-1.19	-2.49	-3.66	3.14	1.70	3.29	3.33	2.92
4	-1.93	-2.28	-3.16	2.51	0.32	1.49	0.21	2.20
5	-2.21	-0.79	-3.29	2.55	2.44	1.45	2.68	3.03
6	-4.14	-2.91	-1.64	3.21	0.37	1.93	0.14	1.27
7	0.21	-1.36	-0.44	2.22	1.85	3.11	2.03	0.67
8	1.13	0.79	2.25	3.65	2.52	2.09	1.13	-2.59
9	0.95	2.33	-0.07	3.89	2.72	2.13	1.75	-2.17
10	3.04	1.85	0.21	7.07	2.01	3.05	0.76	-2.58
11	-1.02	1.65	1.53	0.95	0.60	3.12	2.52	-0.77
12	1.21	0.24	1.04	2.50	3.69	1.81	3.98	-0.33
13	1.74	1.60	1.70	2.02	3.45	4.46	2.69	0.41
14	1.34	1.06	0.06	1.81	2.90	3.64	3.04	0.49
15	0.57	1.81	-0.47	1.40	2.70	0.99	0.82	-1.61
16	0.61	4.22	-2.03	-2.61	-4.00	-4.64	-2.92	1.55
17	-1.13	1.64	0.01	-1.77	-2.85	-1.24	-3.41	-0.59
18	-0.86	-1.17	-0.41	-2.20	-1.30	-2.37	-1.41	0.08
19	0.75	0.66	1.04	-4.26	-1.41	-3.99	-3.53	-2.17
20	0.15	0.68	3.18	-2.86	-2.01	-3.18	-1.58	0.10

Remaining data ($n \times (p-1)$)



Obtain Functional Categories (Annotation)

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MIPS: the Munich Information Center for Protein Sequences

- <http://mips.gsf.de/>
- MIPS: a database for protein sequences and complete genomes, *Nucleic Acids Research*, 27:44-48, 1999

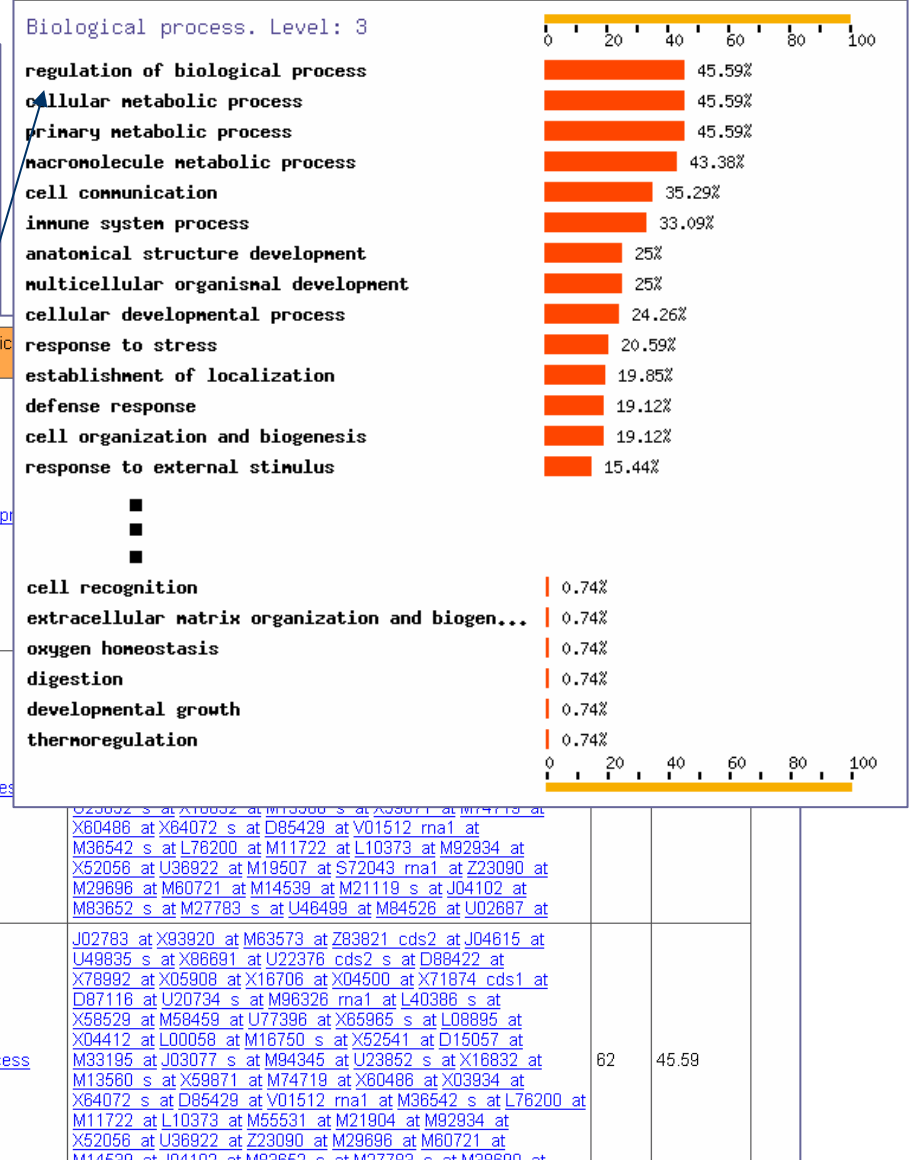
The screenshot shows the MIPS website homepage. The browser title is "Welcome to mips - Mozilla Firefox" and the address bar shows "http://mips.gsf.de/". The page features the MIPS logo and a navigation menu with items like "Projects", "Services", and "About/Contact". A "News" section is visible, containing two articles: "Insights from the genome of the biotrophic fungal plant pathogen Ustilago maydis" and "Anamox in the genes".

The screenshot shows the Gene Ontology website homepage. The browser title is "the Gene Ontology - Mozilla Firefox" and the address bar shows "http://www.geneontology.org/". The page features the Gene Ontology logo and a search bar. A "Search the Gene Ontology Database" section is prominent, with a search input field and a "GO!" button. Below this, there is a "GO website" section with a list of links and information.

GO: Gene Ontology

- A GO annotation is a Gene Ontology term associated with a gene product.
- <http://www.geneontology.org/>
- The Gene Ontology Consortium. *Gene Ontology: tool for the unification of biology. Nature Genet.* (2000) 25: 25-29.
- FatiGO (Al-Shahrour et al., 2004)
- FunCat (Ruepp et al., 2004)

<http://babelomics.bioinfo.cipf.es/index.html>



The ontologies are used to categorize gene products.

- ◆ Biological process ontology
- ◆ Molecular function ontology
- ◆ Cellular component ontology

Software

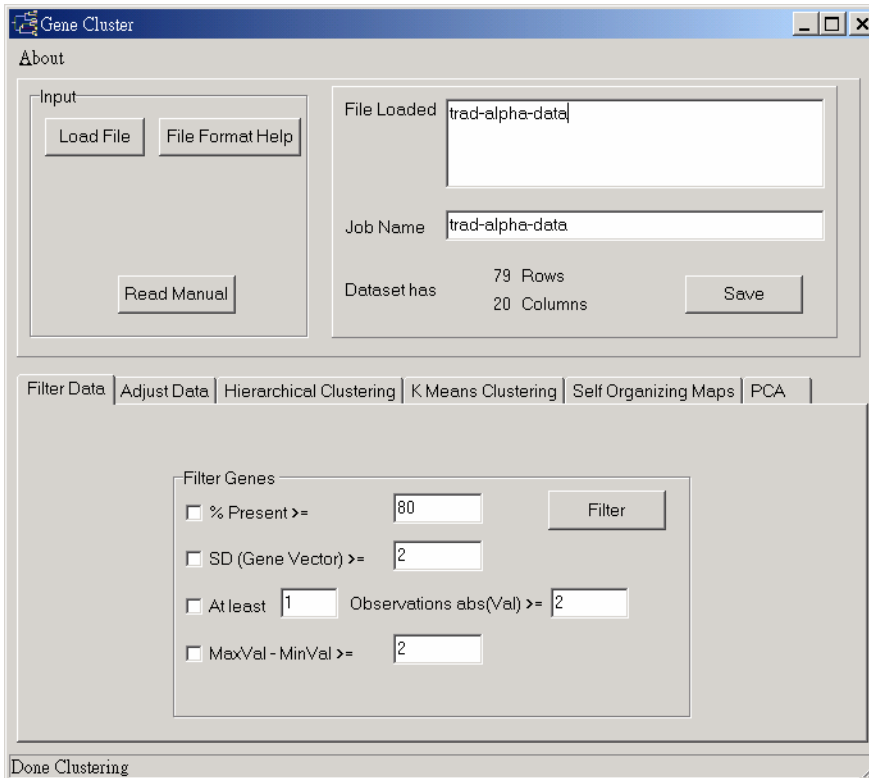
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- Cluster and TreeView
- Bioconductor
- PermutMatrix
- GAP (Generalized Association Plots)

- GeneSpring GX v7.3

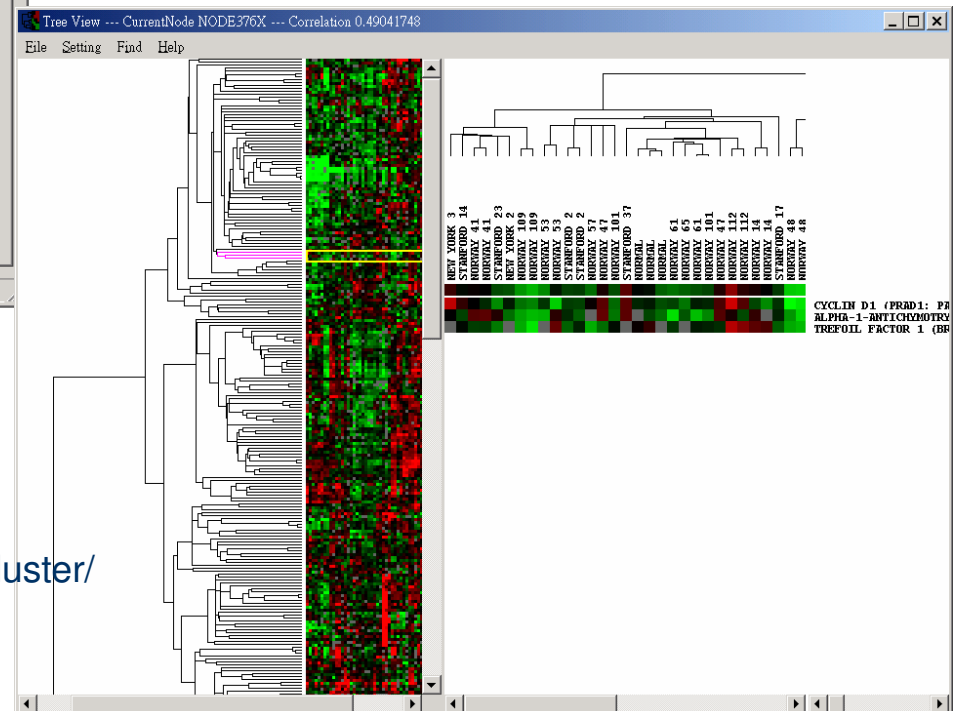
Cluster and TreeView

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<http://rana.lbl.gov/EisenSoftware.htm>

Eisen MB, Spellman PT, Brown PO, Botstein D. (1998) **Cluster analysis and display of genome-wide expression patterns.** *Proc Natl Acad Sci.* 95(25):14863-8.



De Hoon, M.J.L.; Imoto, S.; Nolan, J.; Miyano, S.; **"Open source clustering software"**. *Bioinformatics*, 20 (9): 1453--1454 (2004)

<http://bonsai.ims.u-tokyo.ac.jp/~mdehoon/software/cluster/>

Bioconductor

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Package

[AnnBuilder](#)

[Biobase](#)

[DynDoc](#)

[MAGEML](#)

[MeasurementError.cor](#)

[RBGL](#)

[ROC](#)

[RdbiPgSQL](#)

[Rdbi](#)

[Rgraphviz](#)

[Ruuid](#)

[genefilter](#)

[genepLOTter](#)

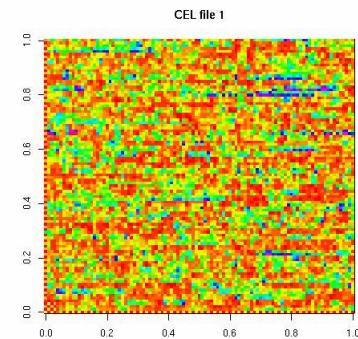
[globaltest](#)

[gpls](#)

[graph](#)

[hexbin](#)

[limma](#)



[daMA](#)

[edd](#)

[externalVector](#)

[factDesign](#)

[gcrma](#)

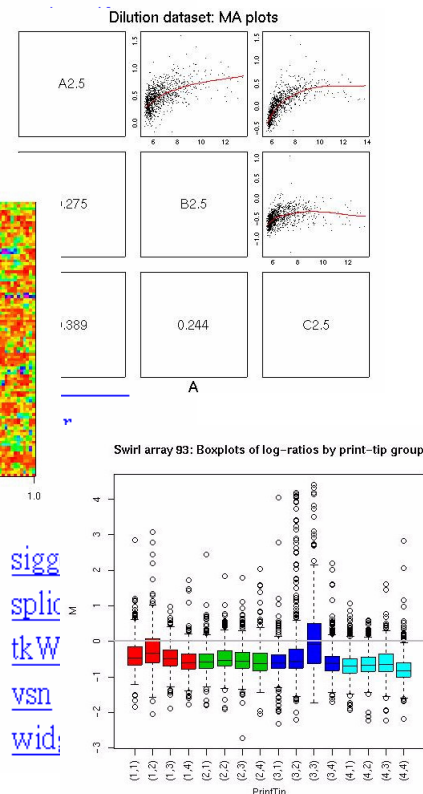
[sigg](#)

[splic](#)

[tkW](#)

[vsn](#)

[wid](#)



The Bioconductor

version 2.0

<http://www.bioconductor.org>



The R Project for
Statistical Computing

R version 2.5.1 (2007-06-28)

<http://www.r-project.org>

RGui

File Edit Misc Packages Windows Help

Load package...
Install package(s) from CRAN...
Install package(s) from local zip files...
Update packages from CRAN
Install package(s) from Bioconductor...
Update packages from Bioconductor

Select

- AnnBuilder
- Biobase
- DynDoc
- MAGEML
- MeasurementError.cor
- RBGL
- ROC
- RdbiPgSQL
- Rdbi
- Ruuid
- Ruuid
- SAGElyzer
- SNPtools
- affyPLM
- affy
- affycomp
- affydata
- annaffy
- annotate

OK Cancel

R 1.8.1 - A Language and Environment

Gclus, PermutMatrix

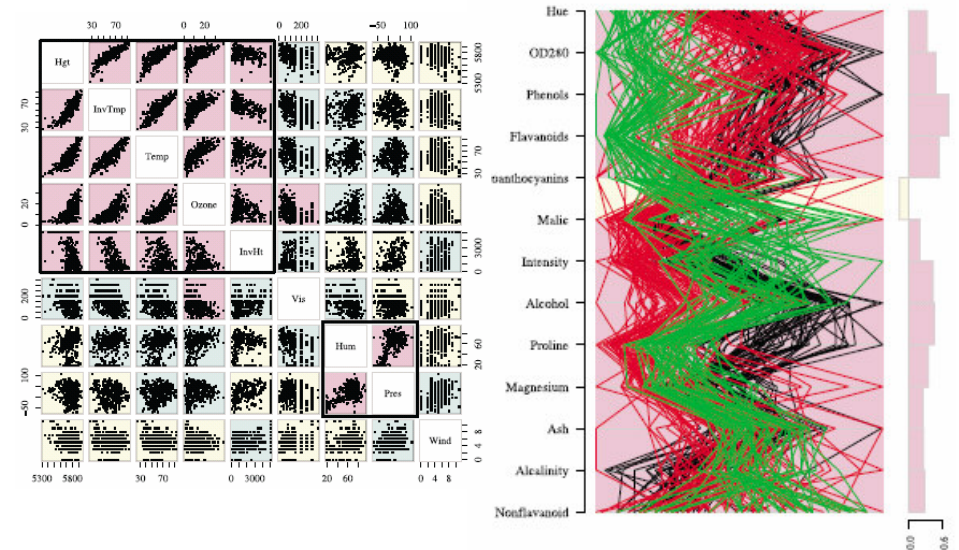
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■ gclus: Clustering Graphics

(R package)

<http://cran.r-project.org/src/contrib/Descriptions/gclus.html>

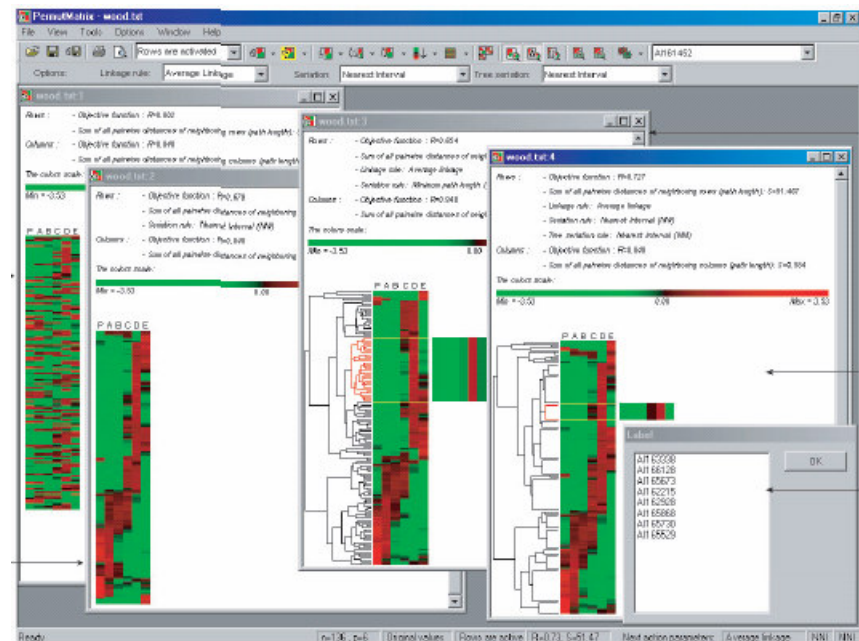
Catherine B. Hurley, (2004), Clustering Visualizations of Multidimensional Data, Journal of Computational & Graphical Statistics, Vol. 13, No. 4, pp.788-806



■ PermutMatrix

<http://www.lirmm.fr/~caraux/PermutMatrix>

Caraux, G., and Pinloche, S. (2005), "Permutmatrix: A Graphical Environment to Arrange Gene Expression Profiles in Optimal Linear Order," Bioinformatics, 21, 1280-1281.



GAP Software version 0.2

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Generalized Association Plots

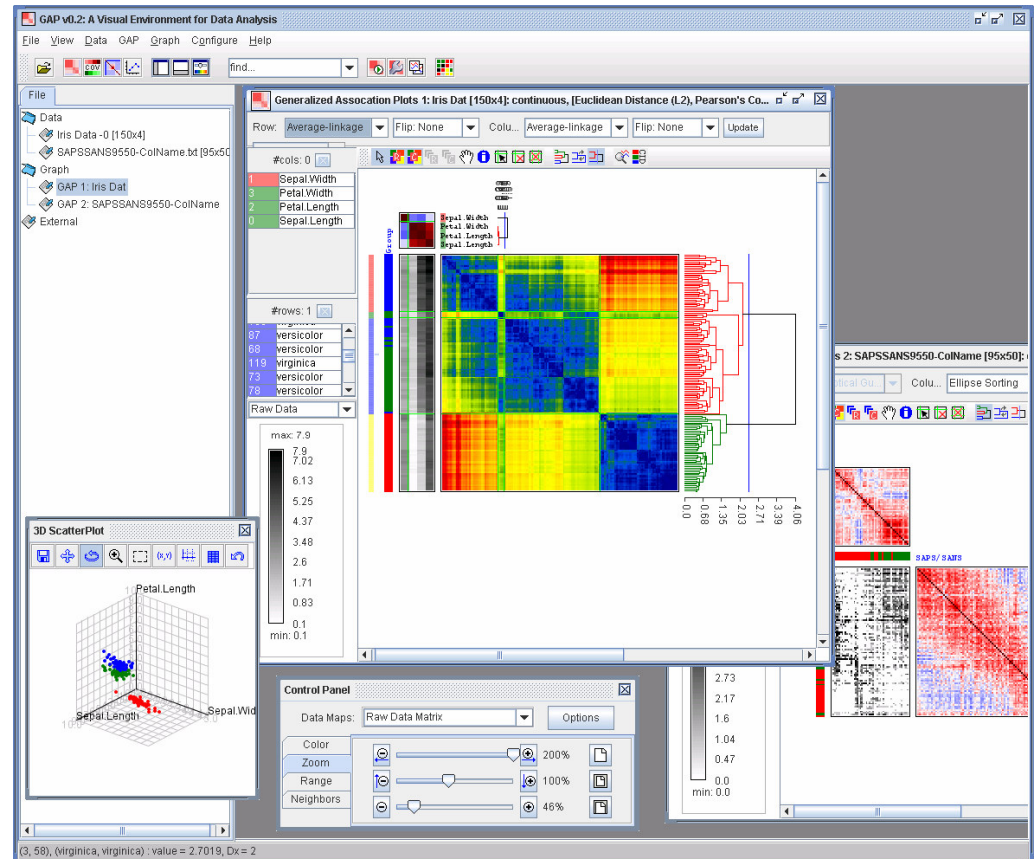
- Input Data Type: continuous or binary.
- Various seriation algorithms and **clustering analysis**.
- Various display conditions.
- Modules:
GAP with Covaraite Adjusted,
Nonlinear Association Analysis,
Missing Value Imputation.

Statistical Plots

- 2D Scatterplot, 3D Scatterplot (Rotatable)

Chen, C. H. (2002). Generalized Association Plots: Information Visualization via Iteratively Generated Correlation Matrices. *Statistica Sinica* 12, 7-29.

Wu, H. M., Tien, Y. J. and Chen, C. H. (2006). GAP: a Graphical Environment for Matrix Visualization and Information Mining.



<http://gap.stat.sinica.edu.tw/Software/GAP>

Matlab: Bioinformatics ToolBox

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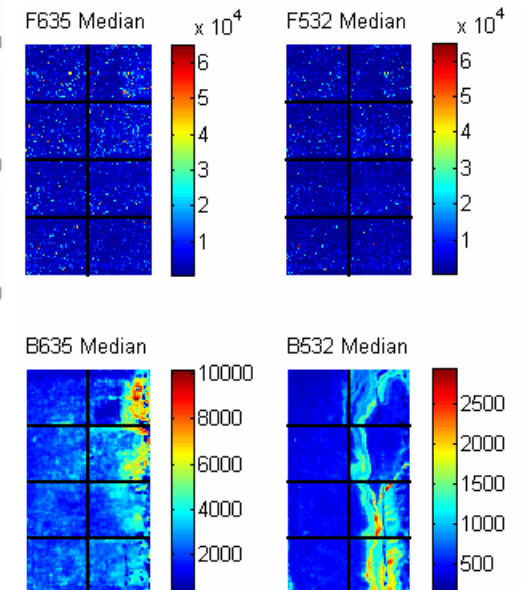
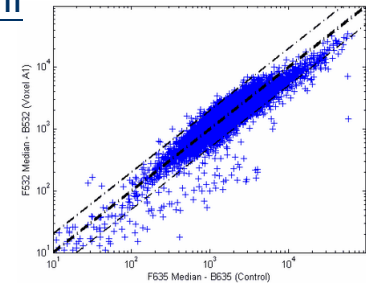
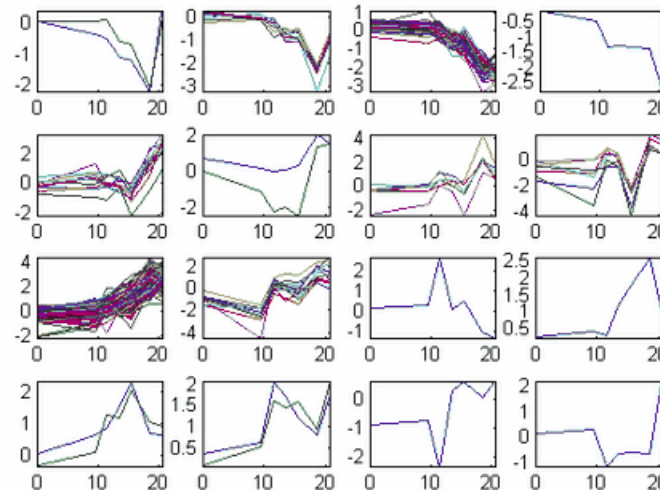


Bioinformatics Toolbox

<http://www.mathworks.com/access/helpdesk/help/toolbox/bioinfo/index.html>

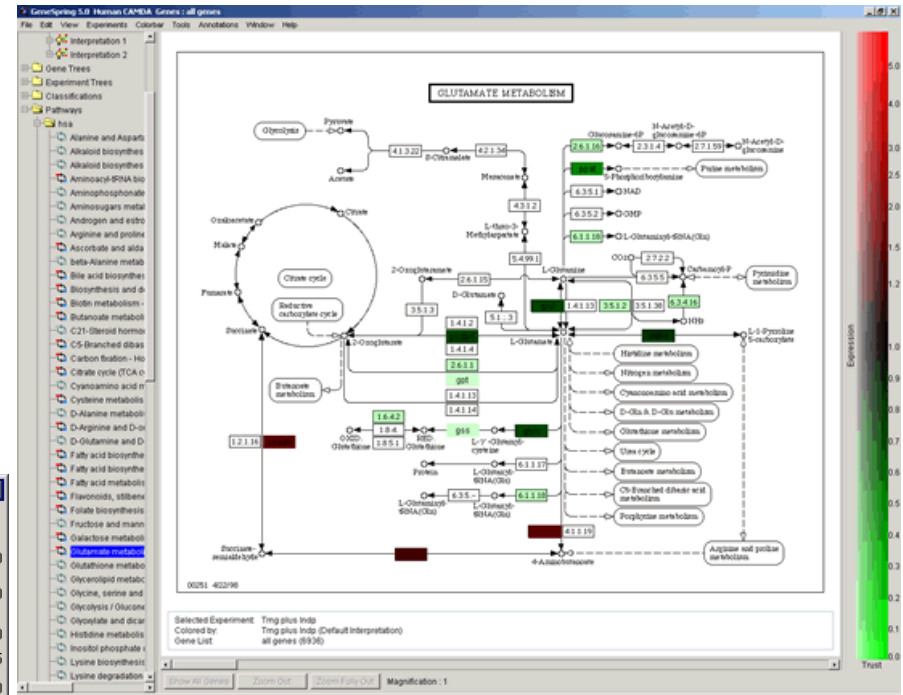
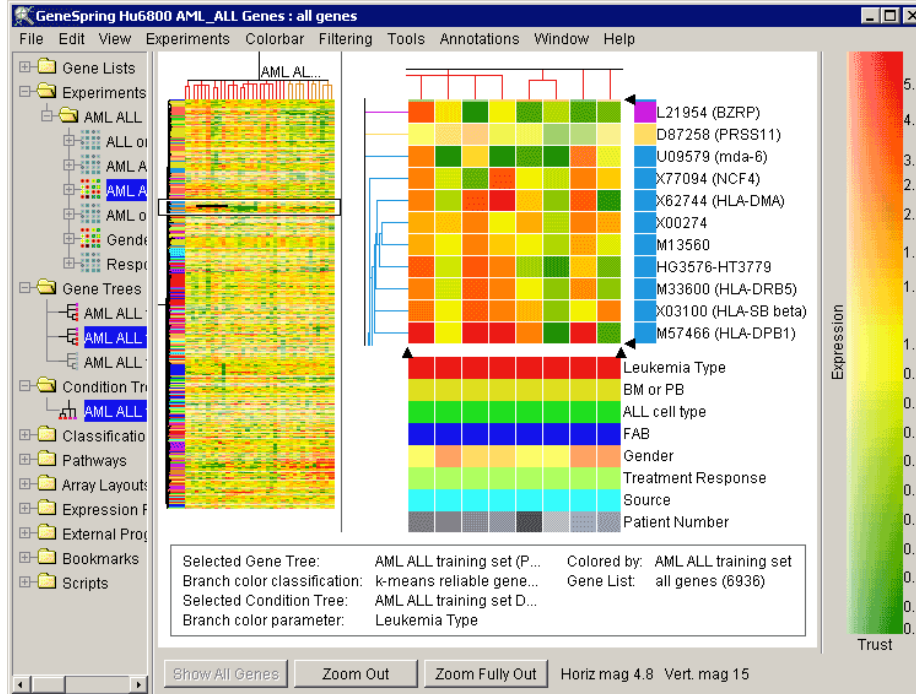
- [Data Formats and Databases](#) — Access online databases, read and write to files with standard genome and proteome formats such as FASTA and PDB.
- [Sequence Alignments](#) — Compare nucleotide or amino acid sequences using pairwise and multiple sequence alignment functions.
- [Sequence Utilities and Statistics](#) — Manipulate sequences and determine physical, chemical, and biological characteristics.
- [Microarray Analysis](#) — Read, filter, normalize, and visualize microarray data.
- [Protein Structure Analysis](#) — Determine protein characteristics and simulate enzyme cleavage reactions.
- [Prototype and Development Environment](#) — Create new algorithms, try new ideas, and compare alternatives.
- [Share Algorithms and Deploy Applications](#) — Create GUIs and stand-alone applications.

Hierarchical Clustering of Profiles



GeneSpring GX v7.3.1

- RMA or GC-RMA probe level analysis
- Advanced Statistical Tools
- Data Clustering
- Visual Filtering
- 3D Data Visualization
- Data Normalization (Sixteen)
- Pathway Views
- Search for Similar Samples
- Support for MIAME Compliance
- Scripting
- MAGE-ML Export



Images from
<http://www.silicongenetics.com>



2004 Articles Citing GeneSpring®

2004 : 2003 : 2002 : 2001 : pre-2001 : Reviews

More than 700 papers

Questions?

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The screenshot shows a Microsoft Internet Explorer browser window displaying the homepage of Hank's Statistical Microarray Data Analysis course. The page title is "Hank's Talks: Statistical Microarray Data Analysis - Microsoft Internet Explorer". The address bar shows the URL: <http://idv.sinica.edu.tw/hmwu/CourseSMDA/index.htm>. The page content includes a navigation menu with links for Home, Experience, Research, Publication, Course, Talks, Software, and Links. A prominent heading reads "Welcome To Hank's Homepage!". Below this, there is a section titled "Talks >> Statistical Microarray Data Analysis" with a sub-section for the year "2007". The list of talks includes:

- [2006/04/21] 3. Design and Analysis for Time Course Microarray Experiments
國防醫學院 生命科學所
- [2006/04/12] 2. Microarray Data Analysis: [Finding Differential Expressed Genes](#) (57pages, 2.741MB)
Case Demo using affyImGUI: <http://bioinf.wehi.edu.au/affyImGUI/>
國立臺灣大學 資訊所, Course: [生物資訊之統計與計算方法](#)
- [2007/03/29] 1. Microarray Data Analysis: Data Preprocessing for Affymatrix GeneChip Data

At the bottom of the screenshot, there are additional links for "Visualization, Clustering and Classification" (33 pages, 3.48MB) and "Gene Regulatory Networks: Bayesian Networks" (24 pages, 1.29MB), with the course name "國立臺灣大學 資訊所 Course: 生物資訊與計算分子生物學".

Thank You!

Reference: <http://idv.sinica.edu.tw/hmwu/SMDA/Clustering/index.htm>

吳漢銘

hmwu@stat.sinica.edu.tw
<http://idv.sinica.edu.tw/hmwu>



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Institute of Statistical Science, Academia Sinica